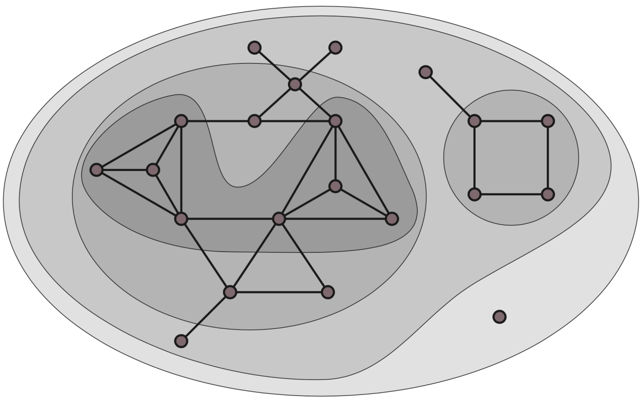
k-core decomposition, node mixing by (not) degree

I. Network k-core decomposition

You are given four networks with some metadata associated with each node.

- Java class dependency network with class names & packages (cdn_java.net)
- JUNG class dependency network with class names & packages (cdn_jung.net)
- WikiLeaks cable reference network with cable identifiers & embassies (wikileaks.net)
- IMDb actors collaboration network with actor names & surnames (collaboration_imdb.net)

1. **(discuss)** Consider the following algorithm for computing network k-cores for a given k. Starting with the original network, iteratively remove nodes with degree less than k. When no such node remains, connected components of the resulting network are the k-cores.



2. **(code)** Implement the algorithm and compute all k-cores of the networks. Print out the number and size of k-cores for different values of k. What is the maximum value of k denoted k_{max} for which there exists at least one k-core?

3. **(code)** Print out the labels of nodes in k_{max}-cores and discuss the results.

II. Degree assortative and disassortative networks

Consider the following eight networks of different type and origin.

- Zachary karate club network (karate_club.net)
- Java class dependency network (java.net)
- Map of Darknet from Tor network (darknet.net)

- Social network of unknown origin (social.net)
- iMDB actors collaboration network (collaboration_imdb.net)
- Gnutella peer-to-peer sharing network (gnutella.net)
- Sample of Facebook social network (facebook.net)
- *nec* overlay map of the Internet (nec.net)

1. **(code)** Implement Newman's node degree mixing coefficient r as a sample Pearson correlation coefficient between the linked nodes' degrees k and k'.

$$r(k,k') = \frac{\sum_i (k_i - \langle k \rangle)(k'_i - \langle k' \rangle)}{\sigma_k \sigma_{k'}}$$

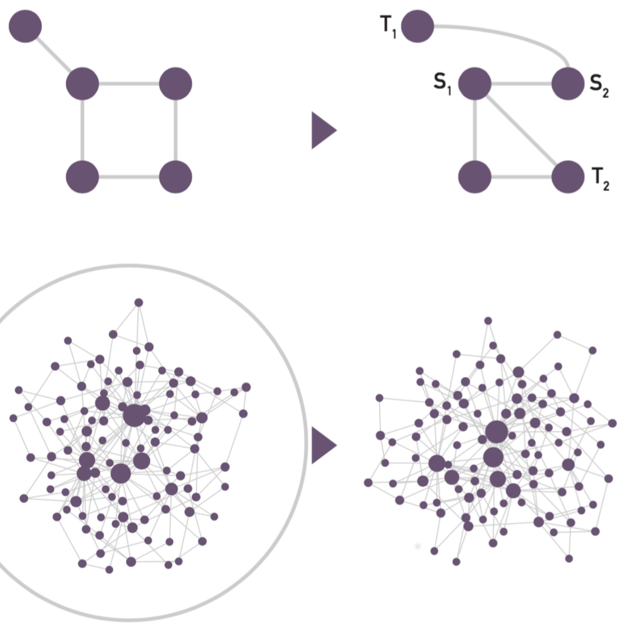
   Treat all networks as undirected graphs and compute their undirected degree mixing coefficient r. Are the networks assortative $r > 0$, disassortative $r < 0$ or neutral $r \approx 0$?

2. **(code)** Generate corresponding Erdös-Rényi random graphs and compute their undirected degree mixing coefficient r. Are random graphs assortative $r > 0$, disassortative $r < 0$ or neutral $r \approx 0$?

3. **(code)** For directed networks, compute all four directed degree mixing coefficients $r_{(in,in)}$, $r_{(in,out)}$, $r_{(out,in)}$ and $r_{(out,out)}$. Are the networks assortative $r_. > 0$, disassortative $r_. < 0$ or neutral $r_. \approx 0$?

III. Structurally disassortative networks by degree

1. **(discuss)** Consider network randomization by degree-preserving link rewiring. What is the expected undirected degree mixing coefficient r' after rewiring if you allow for multiple links between the nodes? What about if you restrict the process to generate only simple graphs?



2. **(code)** Apply link rewiring restricted to simple graph to degree disassortative networks and compute their degree mixing coefficient r' after rewiring. Are the networks *truly* degree disassortative $r' \approx 0$ or

only structurally disassortative $r' < 0$?

IV. Node mixing by *not* degree

(homework) Study node mixing in networks by some property *other* than node degree k. This can be either some structural property of nodes (e.g., node clustering coefficient C or C^{μ}) or external information associated with each node (e.g., sociological partitioning of nodes in social network or traffic loads in highway networks).