core-periphery structure

introduction to *network analysis* (*ina*)

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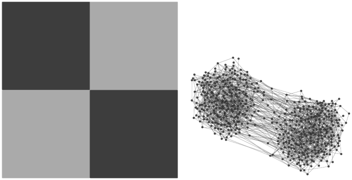
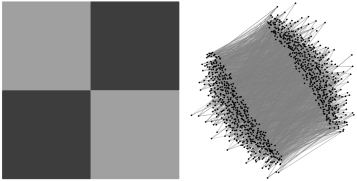
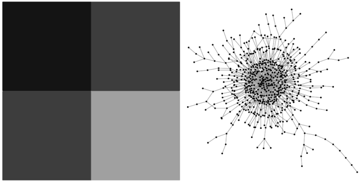
core-periphery *block model*



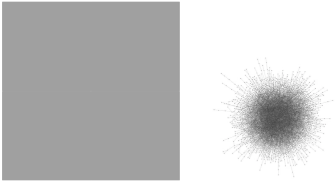
community block model [GN02]



disassortative (bipartite) block model [NL07]



core-periphery block model [Sei83]



random block model [ER59]

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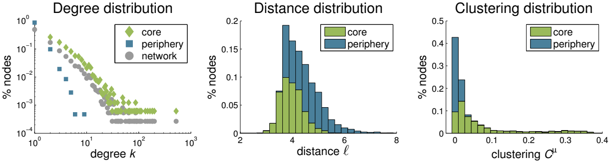
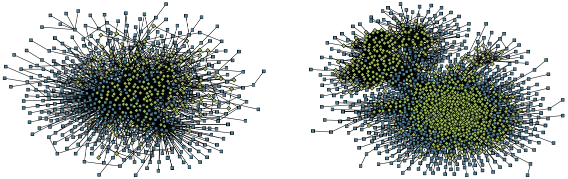
*origin of core-periphery structure in international relations

core-periphery *structure*

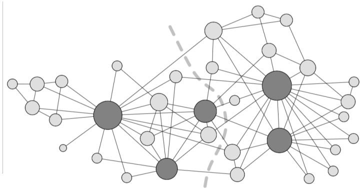
- *core*/*periphery* *nodes* have *higher*/*lower* *degrees* k
- *core*/*periphery* *nodes* are on *shorter*/*longer* *distances* ℓ
- *core*/*periphery* *nodes* have *higher*/*lower* *clustering* C^μ



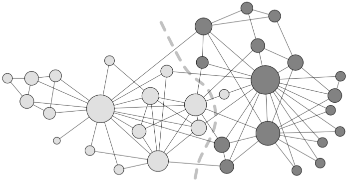


core-periphery *stochastic*

- $G(\{C_1, C_2\}, \{p_{11}, p_{12}, p_{22}\})$ *stochastic block model* [HLL83]
  - n_i is *size* of *cluster* C_i & p_{ij} is *link density* between C_i and C_j
- *density-based core-periphery* structure when $p_{11} \gg p_{12} \gg p_{22}$
- *lookalike core-periph.* when $n_1 p_{11} \gg 1$, $n_1 p_{12} \ll 1$, $n_2 p_{22} \approx 1$



non-corrected block model $p_{11} > p_{12} > p_{22}$



degree-corrected block model $p_{11} \approx p_{22} > p_{12}$

core-periphery *discrete/continuos*

- *discrete* *core-periphery division* $\delta \in \{0, 1\}$ [BE00]
  - $\delta_i = 1$ for *core nodes* i & $\delta_i = 0$ for *peripheral nodes* i

$$\rho_{\{0,1\}} = \sum_{ij} A_{ij} \Delta_{ij}^{\alpha} \qquad \Delta_{ij} = \begin{cases} 1 & \text{if } \delta_i = \delta_j = 1 \\ 0 & \text{if } \delta_i = \delta_j = 0 \\ \alpha \in [0, 1] & \text{if } \delta_i - \delta_j \neq 0 \end{cases}$$

- *continuos* *core-periphery centrality* $\delta \in [0, 1]$
  - $\delta_i \approx 1$ for *core nodes* i & $\delta_i \approx 0$ for *peripheral nodes* i

$$\rho_{[0,1]} = \sum_{ij} A_{ij} \delta_i \delta_j$$

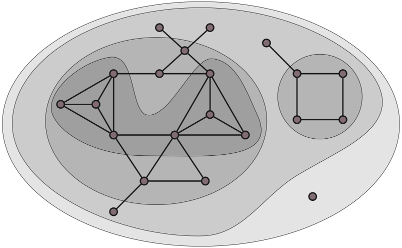
$$\Delta^1 = \left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{array}\right] \quad \Delta^\alpha = \left[\begin{array}{ccc|ccc} 0 & 1 & 1 & \alpha & \alpha & \alpha \\ 1 & 0 & 1 & \alpha & \alpha & \alpha \\ 1 & 1 & 0 & \alpha & \alpha & \alpha \\ \hline \alpha & \alpha & \alpha & 0 & 0 & 0 \\ \alpha & \alpha & \alpha & 0 & 0 & 0 \\ \alpha & \alpha & \alpha & 0 & 0 & 0 \end{array}\right] \quad \delta = \left[\begin{array}{c} 1 \\ 0.8 \\ 0.7 \\ \hline 0.4 \\ 0.2 \\ 0.1 \end{array}\right]$$

core-periphery *k-cores*

- *k-cores* are *subgraphs of nodes* with $\geq k$ *neighbors* [Sei83]

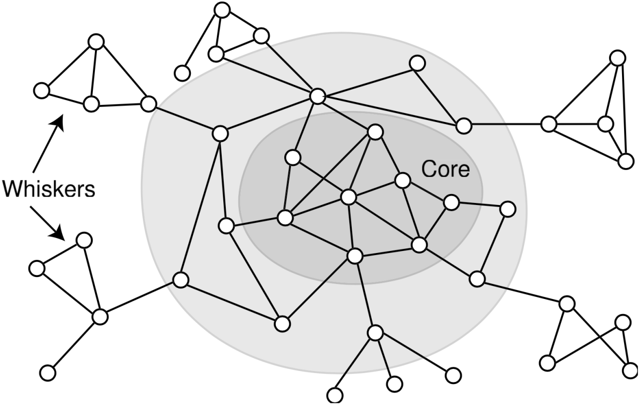
  remove nodes with degree $< k$ until no such node remains [BZ11]

- *k-shells* are *nodes of k-cores* that are *not in $k+1$-cores*
- *k-cores* are *nested* while *k-shells* form *decomposition*



1-cores are connected components w/o isolates & k-cores can be disconnected

core-periphery *nestedness*



nested cores & whiskers communities [LLDM09, YL13]

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core-periphery *references*

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