node *mixing*

introduction to *network analysis* (*ina*)

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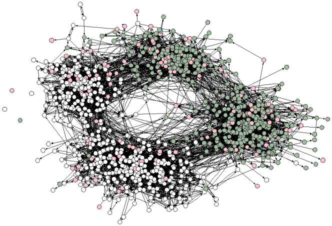
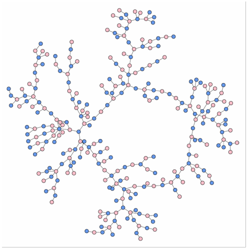
mixing *definition*

- *node mixing* = *correlations between linked* nodes
- in *assortative mixing* nodes are *linked to similar* others
- in *disassortative mixing* nodes *linked to dissimilar* others



assortative mixing by age/race



disassortative mixing by gender

mixing *degree*

- special case of *node mixing* *by degree* [New02]
- majority of *social networks* *degree* *assortative*
- most *other networks* are *degree* *disassortative*

$$p_{kk'} = k\frac{k'}{2m-1} = m\frac{kk'}{\binom{2m}{2}} \approx \frac{kk'}{2m}$$

celebrity hubs date hubs

but $10^3/10^8 = 0.00001$

protein hubs avoid hubs

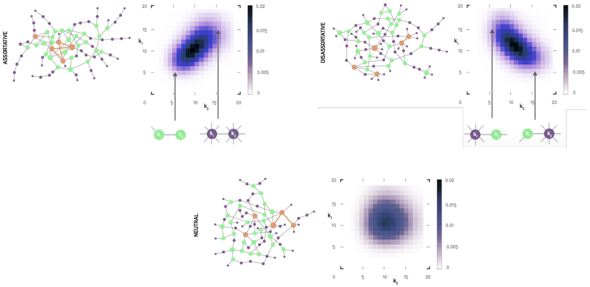
but $p_{56,13} = 0.16 \gg p_{1,2} = 0.0004$

mixing *matrix*

- *endpoints degree distribution* $e_{kk'}$ defined as
  - $e_{kk'}$ is *link probability* between *degree-k & -k'* nodes
  - q_k is *neighbor non-excess degree distribution* $\frac{kp_k}{\langle k \rangle}$

$$\sum_{kk'} e_{kk'} = 1 \qquad\qquad \sum_{k'} e_{kk'} = q_k = n_k \frac{k}{2m-1} \approx \frac{kp_k}{\langle k \rangle}$$

$e_{kk'} = q_k q_{k'}$ in *neutral networks* but impractical for (*dis*)*assortative networks*



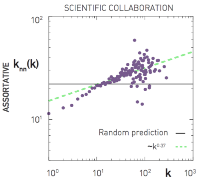
mixing *exponent*

— *neighbor degree function* k_{nn} [PSVV01] defined as

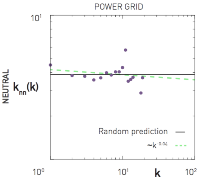
- $k_{nn}(k)$ is *average neighbor degree* of *degree-k* nodes
- $\mathrm{P}(k'|k)$ is *link probability* of *degree-k to -k'* node
- μ is *degree mixing power-law exponent* [VPSV02]

$$k_{nn}(k) = \sum_{k'} k' \mathrm{P}(k'|k) = \sum_{k'} k' \frac{e_{kk'}}{\sum_{k'} e_{kk'}}$$

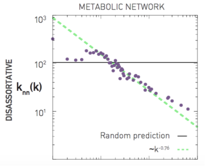
$k_{nn} = \frac{\langle k^2 \rangle}{\langle k \rangle}$ in *neutral networks* and $k_{nn}(k) \sim k^{\mu}$ in (*dis*)*assortative networks*



$\mu = 0.37 \pm 0.11$



$\mu = -0.04 \pm 0.05$



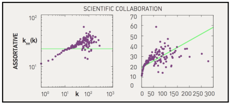
$\mu = -0.76 \pm 0.04$

mixing *coefficient*

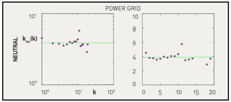
- *degree mixing coefficient* r [New02, Est11] defined as
  - r is *Pearson correlation* of *linked nodes' degrees* [New03]
  - q_k is *neighbor excess degree distribution* $\frac{(k+1)p_{k+1}}{\langle k \rangle}$

$$r = \sum_{kk'} \frac{kk'(e_{kk'} - q_k q_{k'})}{\sum_k k^2 q_k - \left(\sum_k k q_k\right)^2}$$

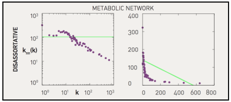
$r = 0$ in *neutral networks* and $k_{nn}(k) \sim rk$ in *(dis)assortative networks*



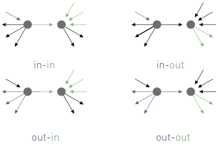
$r = 0.13$

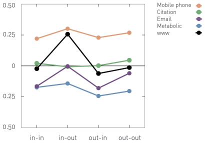


$r = 0$



$r = -0.04$



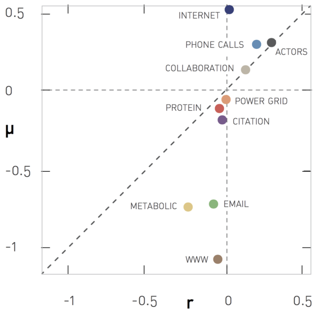


mixing *networks*

— *coefficient & exponent* r *&* μ in real networks [Bar16]

— r *&* μ *correlate* in *assortative* regime and $\mathrm{sgn}(r) = \mathrm{sgn}(\mu)$

NETWORK	N	r	μ
Internet	192,244	0.02	0.56
WWW	325,729	-0.05	-1.11
Power Grid	4,941	0.003	0.0
Mobile Phone Calls	36,595	0.21	0.33
Email	57,194	-0.08	-0.74
Science Collaboration	23,133	0.13	0.16
Actor Network	702,388	0.31	0.34
Citation Network	449,673	-0.02	-0.18
E. Coli Metabolism	1,039	-0.25	-0.76
Protein Interactions	2,018	0.04	-0.1



mixing *structural*

— *structural disassortativity* $\frac{E_{kk'}}{m_{kk'}} > 1$ [MSZ04] in real networks

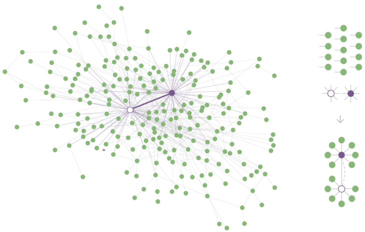
  – $E_{kk'}$ is *number of links* between *degree-k & -k'* nodes

  – $m_{kk'}$ is *maximum* $E_{kk'}$ hence $\min(kn_k, k'n_{k'}, n_k n_{k'})$

$$E_{kk'} = 2me_{kk'} = \langle k \rangle n e_{kk'}$$

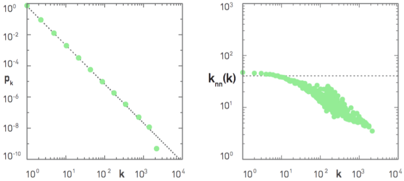
natural cutoff $k_{max} \sim n^{\frac{1}{\gamma-1}}$ and *structural cutoff* $k_s \sim \sqrt{\langle k \rangle n}$

— *structural disassortativity* in *scale-free* networks with $\gamma < 3$

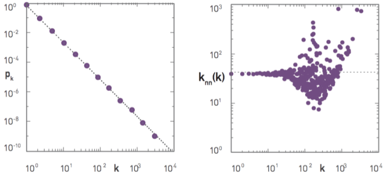


$k = 55$ and $k' = 46$ then $E_{kk'} = 2.81 > 1$

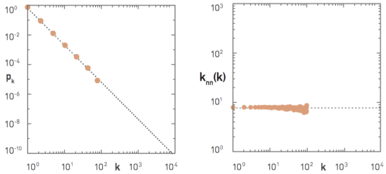
mixing *scale-free*



configuration *scale-free* network as *simple graph*

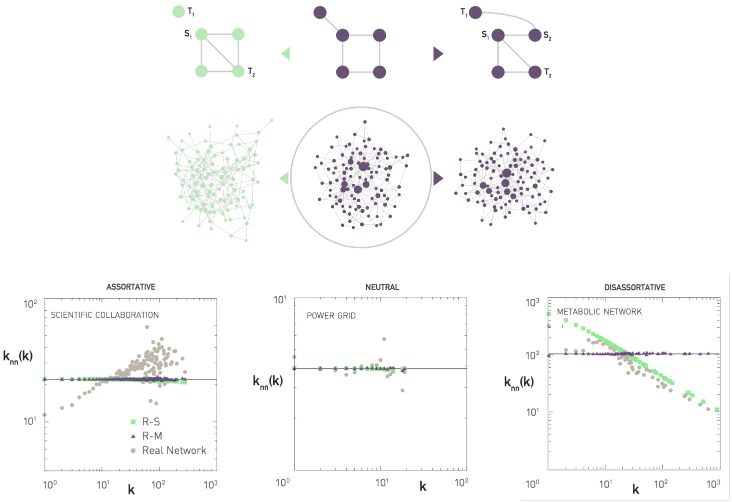


configuration *scale-free* network as *multigraph*



configuration *scale-free* network *without hubs* $k \geq k_s$

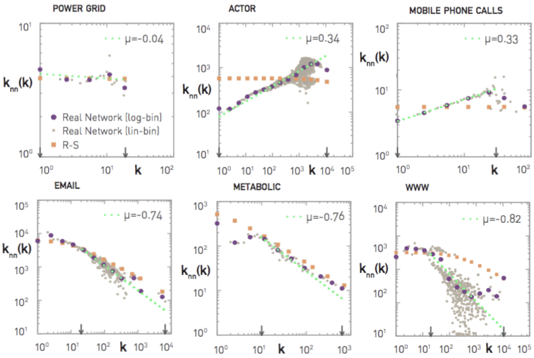
mixing *randomization*



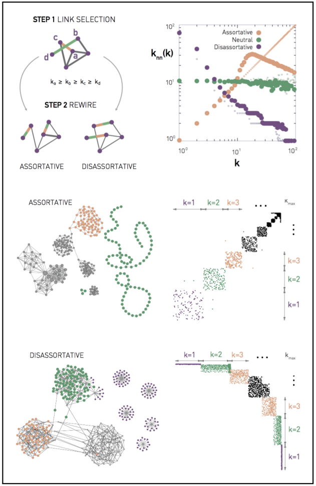
degree-preserving randomization with *simple/multi* links *retains/destroys structural disassortativity*

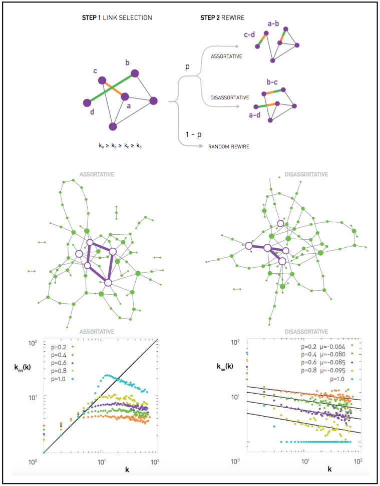
mixing *networks*

- *neighbor degree* k_{nn} in real networks [Bar16]
- *collaboration assortative* and *technological neutral*
- *biological/information (structurally) disassortative*



mixing *models*

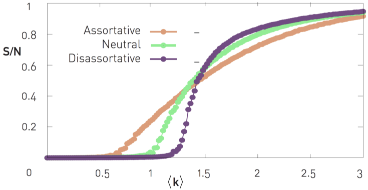


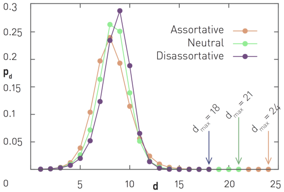


(dis)assortative degree-preserving randomization [XBS05]

mixing *impact*

- *degree mixing* impacts *connectivity* and *distances* [New02]
- *assortative mixing* coexists with *community structure* [NP03]
- *mixing* influences *resilience* [VM03] and *controllability* [LSB11]





mixing *references*

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mixing *references*

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