graph theory $\rightarrow$ *network science*

introduction to *network analysis* (*ina*)

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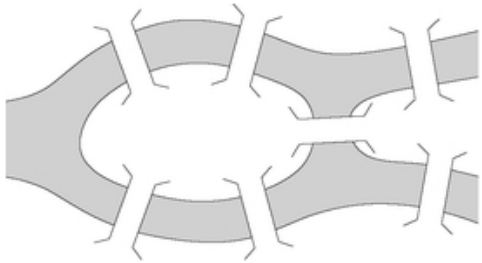
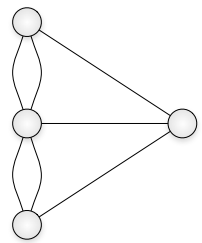
spring 2024/25

history *graph theory*

1736 seven *bridges of Königsberg* [Eul36] (Leonhard Euler)

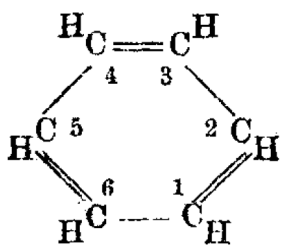
1800s *travelling salesman* problem (William Hamilton)

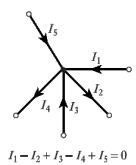


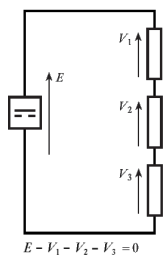


1845 *electrical circuit* laws (Gustav Kirchhoff)

1857 *chemical structure* theory (August Kekulé)







history *operations research*

1956 *shortest paths* (Edsger Dijkstra)

1956 minimum *spanning tree* (Joseph Kruskal)

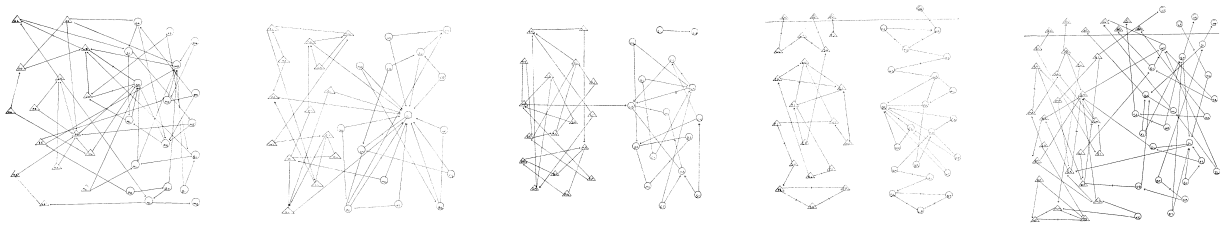
1956 maximum *flow*/minimum *cut* (Ford & Fulkerson)

1956 *signed graph* theory [CH56] (Cartwright & Harary)

1959 *random graph* theory [ER59] (Erdős & Rényi)

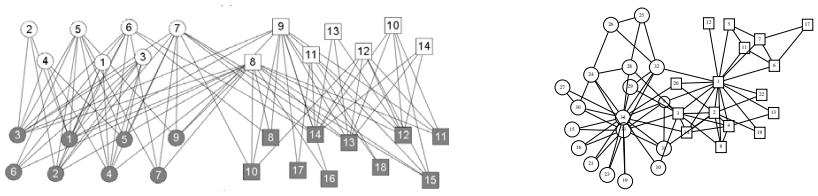
history *sociometry*

1934 children *sociograms* [Mor34] (Jacob Moreno)



1941 *Southern women* [DGG41] (Allison Davis)

1970 university *karate club* [Zac77] (Wayne Zachary)



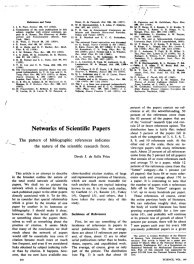
1967 *small-world* experiment [Mil67] (Stanley Milgram)

1973 strength of *weak ties* [Gra73] (Mark Granovetter)

1977 measures of *centrality* [Fre77] (Linton Freeman)

history *bibliometrics*

1965 scientific *paper citations* [Pri65] (Derek de Solla Price)

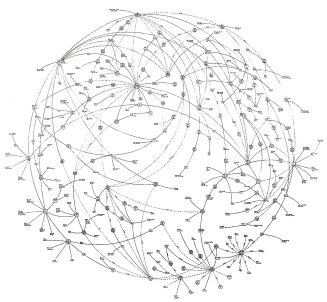

Networks of Scientific Papers

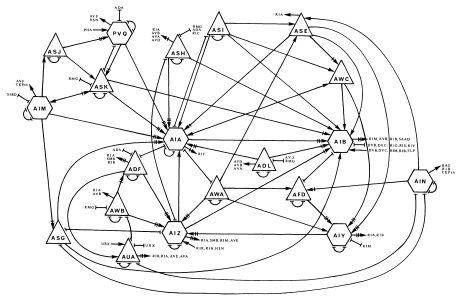
SCIENCE CITATION INDEX

1980s *political scandals* [HL03] (Mark Lombardi)

1986 *neural wirings* [WSTB86] (White et al.)

1999 transportation [Pel99] (Jon Pelletier)





revolution *data*

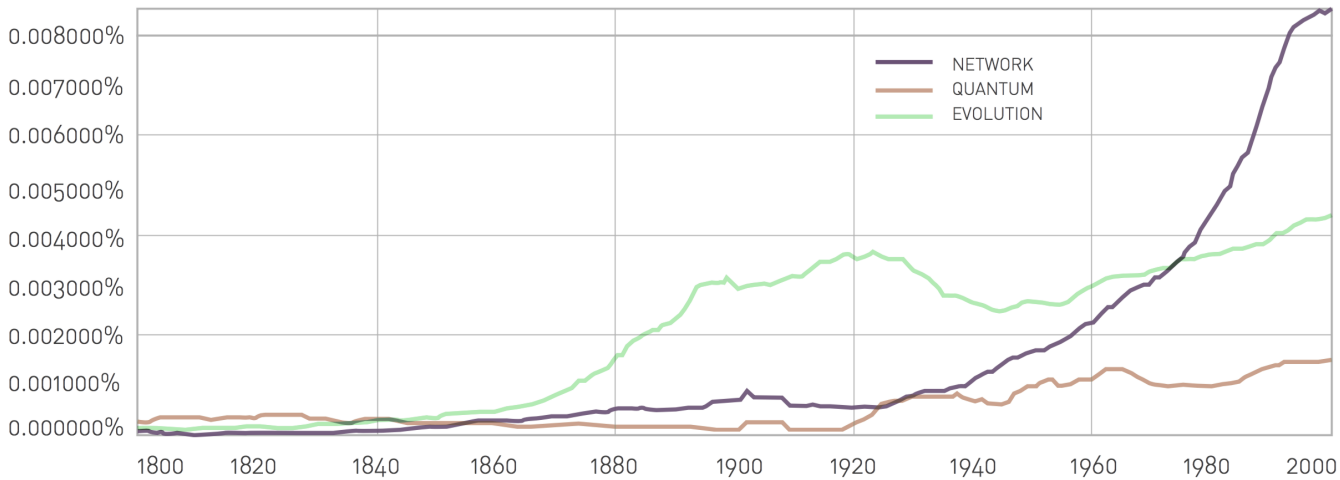
< 2000 *small graphs* 10^2-10^3 nodes

≈ 2000 *communication networks* 10^5-10^8 nodes

≈ 2005 *online social networks* 10^8 nodes

today *Facebook graph* $> 10^9$ users

today *Web graph* $> 10^{12}$ pages



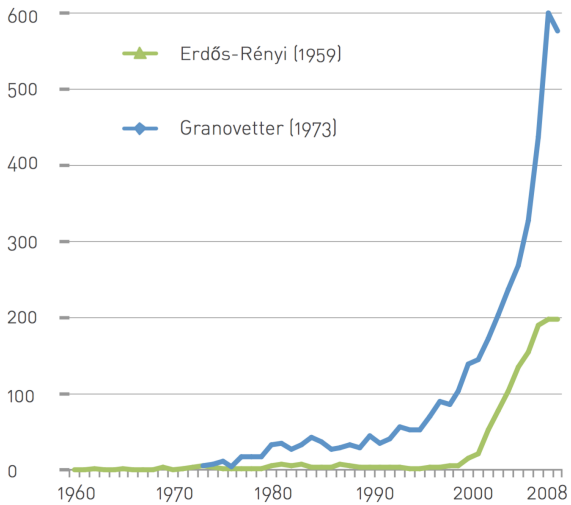
revolution *models*

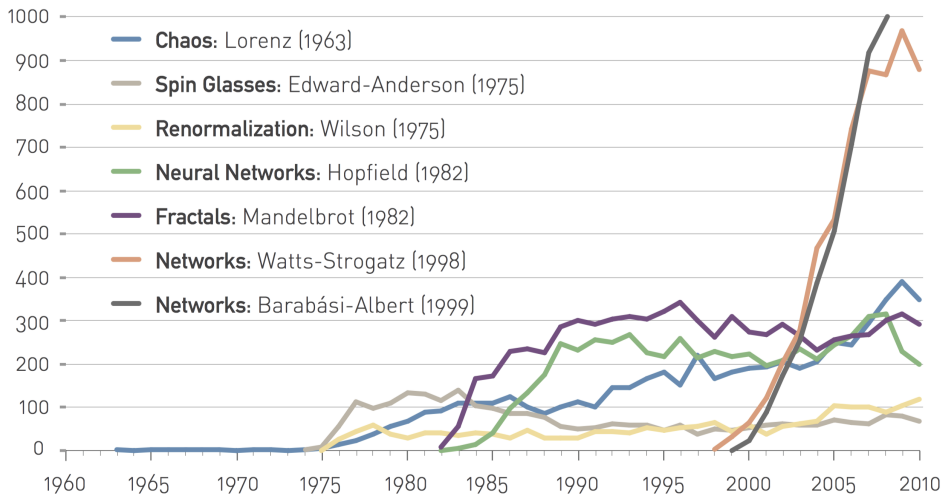
1959 *random graph* models [ER59]

1973 *valued graphs* models [Gra73]

1998 *small-world network* structure [WS98]

1999 *scale-free network* structure [BA99]





revolution *language*

"A key discovery of network science is that the architecture of networks emerging in various domains of science, nature, and technology are similar to each other, a consequence of being governed by the same organizing principles. Consequently we can use a common set of tools to explore these systems."

Albert-László Barabási

"Networks are ideal structures to describe problems of organized complexity."

César A. Hidalgo

"I think the 21st century will be the century of complexity."

Stephen Hawking

network *impact*

- *management:* internal structure of organization
- *economic:* from web search to social networking
- *epidemics:* from forecasting to halting deadly viruses
- *health:* from drug design to metabolic engineering
- *security:* fraud detection and fighting terrorism
- *neuroscience:* mapping human brain
- *many other:* ...

network *science*

problem

understanding *real networks*

means

study of *network properties*

design of *mathematical models*

implementation of *efficient algorithms*

goals

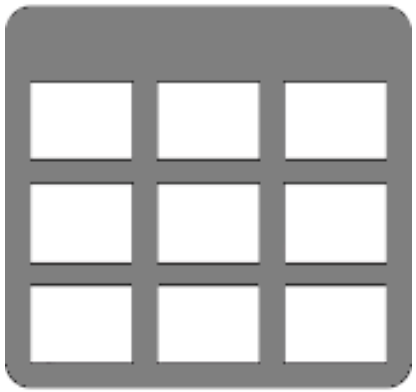
network *structure* and *evolution*

nodes, links, fragments, clusters, layers, networks

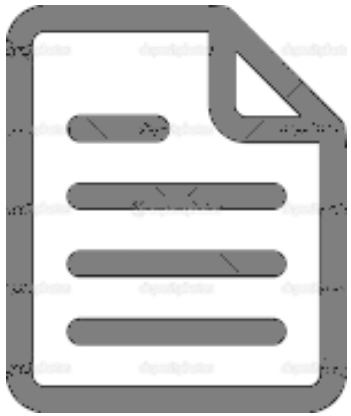
network *dynamics* and *processes*

spreading, diffusion, epidemics

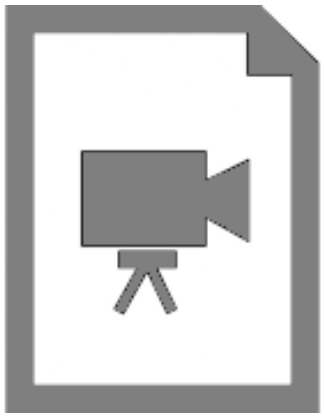
network *analysis*



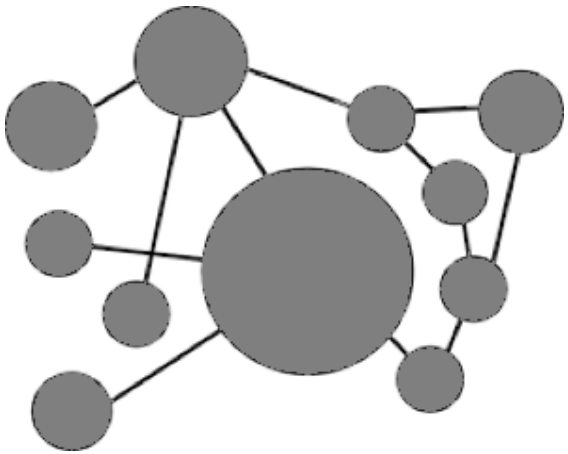
data mining



text mining



computer vision



network analysis

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