

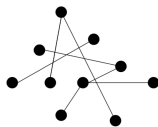
network *sampling*

introduction to *network analysis* (*ina*)

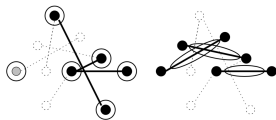
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sampling *overview*

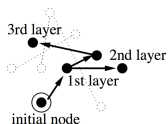
- *snowball sampling* and *contact tracing* in *sociology*
network-based methods for sampling hidden populations
- *fractality* and *self-similarity* of *real networks* [SHM05]
fractality is property of object that it is similar to part of itself
self-similarity demands power-law size scaling under renormalization
- any *real network* is just *sample* of *true network* [BŠWB15]



original network



node/link selection and snowball sampling



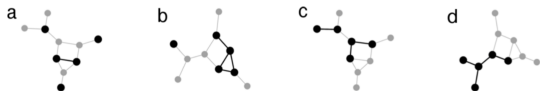
sampling *populations*

- *snowball sampling* similar to *breadth-first search*
nodes sampled proportional to their eigenvector centrality
- *contact tracing* similar to biased *best-first search*
nodes sampled according to some hidden variable
- *respondent-driven sampling* similar to *random walk*
nodes sampled proportional to their degree

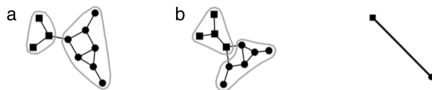
$$\text{estimate } \langle x \rangle = \frac{\sum_i x_i}{\sum_i 1} \quad \text{corrected } \langle x \rangle = \frac{\sum_i x_i / k_i}{\sum_i 1 / k_i}$$

sampling *methods*

- *random selection methods* for *global network sparsification*
random node/link selection w/ or w/o induction etc.
- *network exploration methods* for *local network sampling*
random walks, snowball and expansion sampling etc.
- *merging/aggregation methods* for *network simplification*
box covering, cluster growing, community aggregation etc.



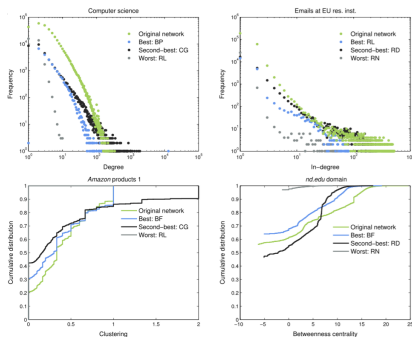
(a) node selection (b) by degree (c) link selection and (d) snowball sampling



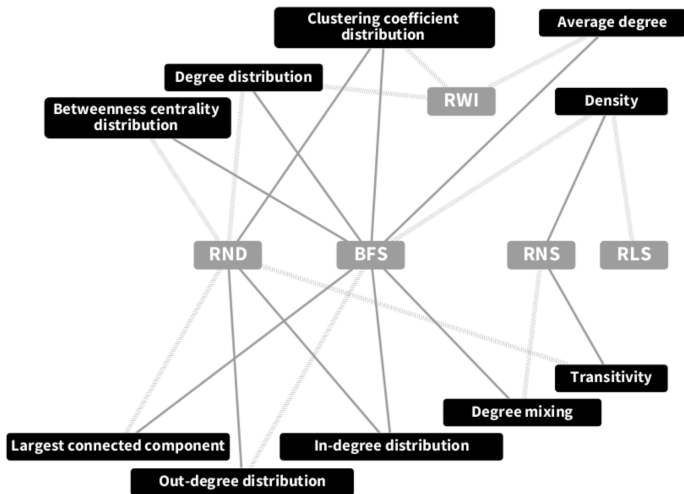
(a) community aggregation and (b) cluster growing

sampling *networks*

- *similar sampled networks* for $\gtrsim 15\%$ *network* [LF06]
 - *methods indistinguishable* for $\ll 1\%$ *network* [BŠB17]
 - *selection/exploration* \ggg *aggregation methods* [BŠB14]
- independent of network type or size but not sample size

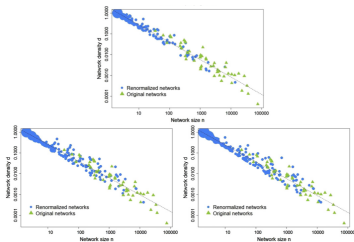


sampling *scheme*

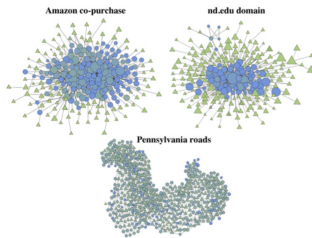


sampling *density*

- *self-similar network density* of *real networks* [LJT⁺11]
most real networks are sparse with $\rho \approx 7.89n^{-0.99}$
- *network density invariant* under *any renormalization* [BŠB12]
box covering, cluster growing, community aggregation etc.



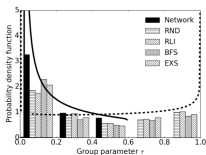
density invariance under renormalization



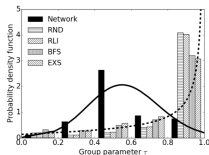
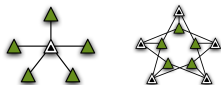
renormalized networks

sampling *communities*

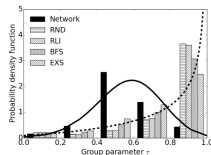
- *pronounced community structure* in *real networks* [GN02]
many real networks contain communities with $\tau \gg 0$
- *community structure enhanced* under *any sampling* [BŠWB15]
random selection, snowball and expansion sampling etc.



citation network



social network



collaboration network



sampling *references*



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sampling *references*



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