

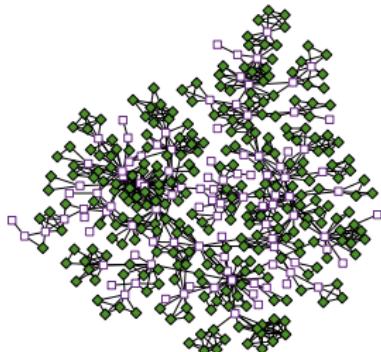
backbones & *skeletons*

introduction to *network analysis* (*ina*)

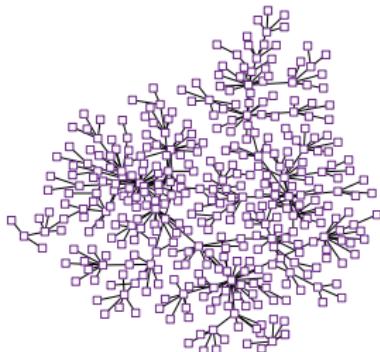
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skeletons *overview*

- *network backbone* retains *strongest links/information flow*
sparsification technique that removes as many links as possible
- *network skeleton* retains *micro/meso/macro structure* [CN17]
simplification technique that retains as many links as possible



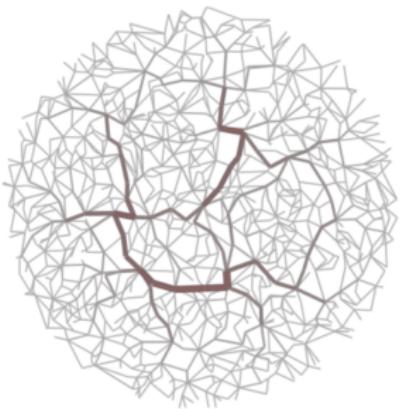
coauthorship network



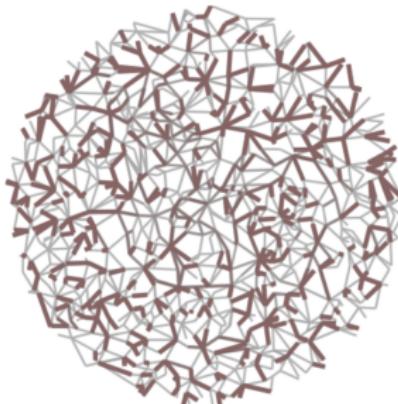
network spanning tree

skeletons *geodesics*

- *betweenness backbone* & *salience skeleton* of graph [GTB12]
betweenness/salience \sim number of geodesics/spanning trees including edge



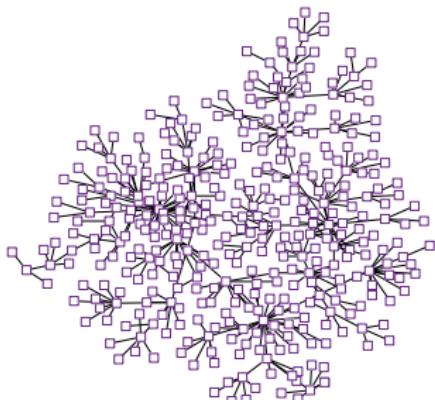
high-betweenness backbone



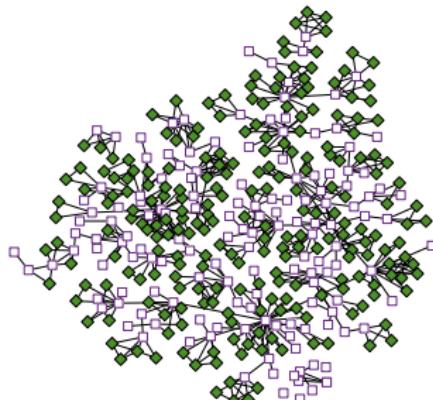
high-salience skeleton

skeletons *convexity*

- *convex skeleton* of famous *network scientists* [MŠ18, Šub18]
largest skeleton where every subgraph includes most geodesics



network spanning tree



convex skeleton \sim tree with cliques

skeletons *statistics*

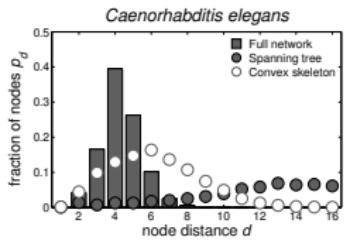
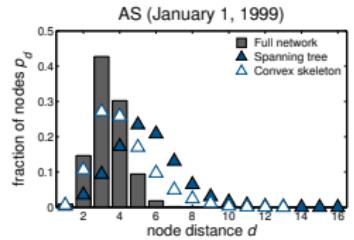
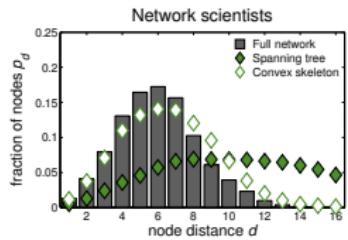
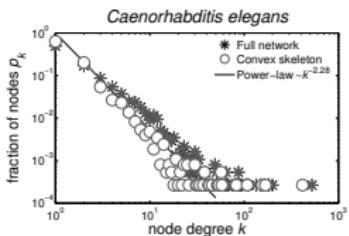
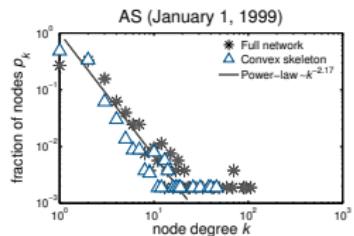
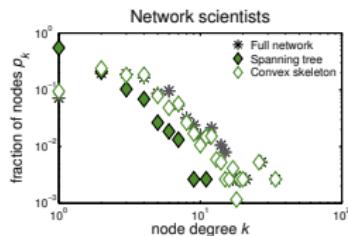
- *large-scale statistics* of *skeletons of networks* [Šub18]
convex skeletons retain connectivity, clustering and geodesics

	clustering $\langle C \rangle$			geodesics $\langle \sigma \rangle$			convexity X_s		
	N	CS	ST	N	CS	ST	N	CS	ST
Jazz musicians	0.62	0.81	0.00	9.71	1.97	1.00	0.12	0.84	1.00
Network scientists	0.74	0.75	0.00	2.66	1.47	1.00	0.85	0.95	1.00
Computer scientists	0.48	0.54	0.00	4.08	1.42	1.00	0.64	0.95	1.00
<i>Plasmodium falciparum</i>	0.02	0.07	0.00	3.71	1.77	1.00	0.43	0.95	1.00
<i>Saccharomyces cerevisiae</i>	0.07	0.10	0.00	2.58	1.19	1.00	0.68	0.88	1.00
<i>Caenorhabditis elegans</i>	0.06	0.12	0.00	6.79	3.03	1.00	0.56	0.85	1.00
AS (January 1, 1998)	0.18	0.21	0.00	3.87	2.32	1.00	0.66	0.91	1.00
AS (January 1, 1999)	0.18	0.27	0.00	3.54	2.05	1.00	0.49	0.95	1.00
AS (January 1, 2000)	0.20	0.25	0.00	4.81	3.07	1.00	0.59	0.90	1.00
Little Rock Lake	0.32	0.69	0.00	22.13	4.32	1.00	0.02	0.82	1.00
Florida Bay (wet)	0.33	0.79	0.00	9.17	1.37	1.00	0.03	0.92	1.00
Florida Bay (dry)	0.33	0.82	0.00	9.37	1.65	1.00	0.03	0.93	1.00

skeletons *distributions*

— node distributions of skeletons of networks [Sub18]

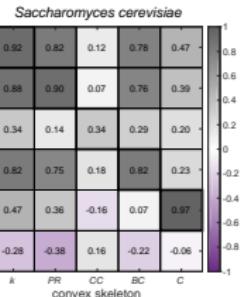
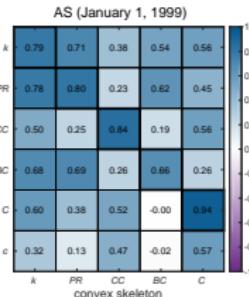
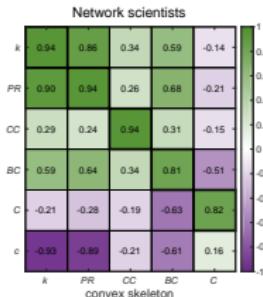
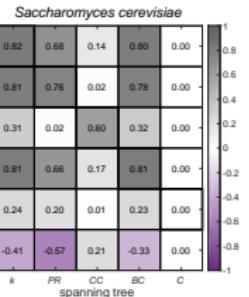
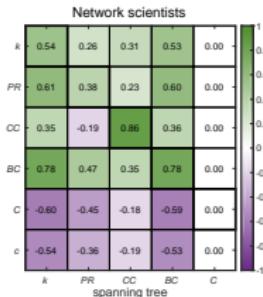
convex skeletons retain node degrees and distances



skeletons *centralities*

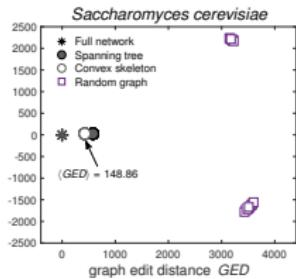
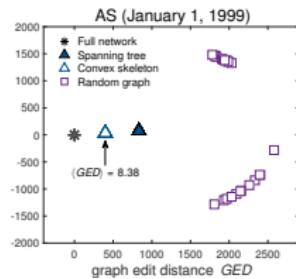
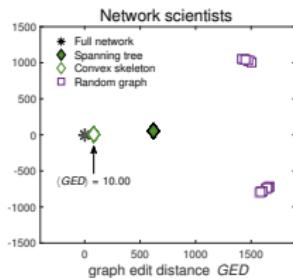
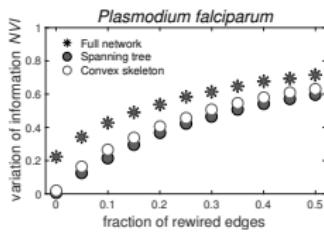
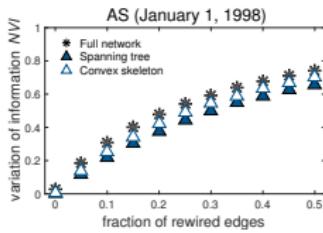
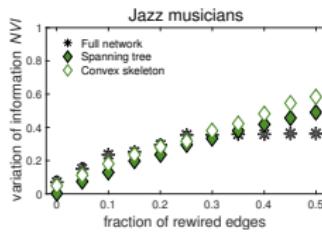
— *node centralities* in *skeletons of networks* [Šub18]

convex skeletons retain node centralities and clustering



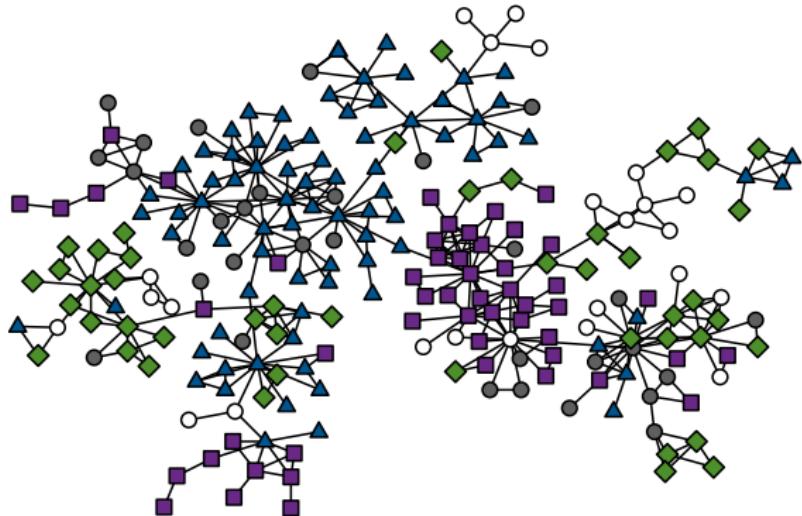
skeletons *communities*

- *communities & robustness of skeletons of networks* [Sub18]
convex skeletons highlight communities with short edit distance



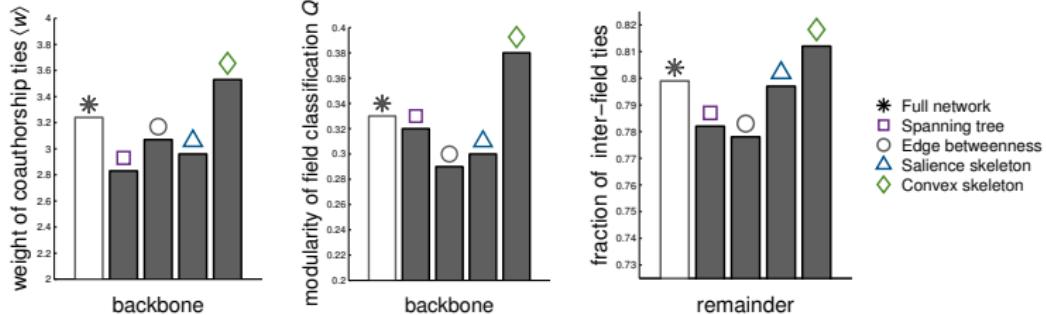
skeletons *coauthorships*

— convex skeleton of SI computer scientists [**Šub18, ŠFCK19**]
computer theory (◆), information syst. (■), intelligent syst. (▲), programming (○) etc.



skeletons *properties*

- properties of *skeletons of coauthorships* [Šub18, ŠFCK19]
convex skeletons retain strong intra-field coauthorships



skeletons *references*

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skeletons *references*

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