

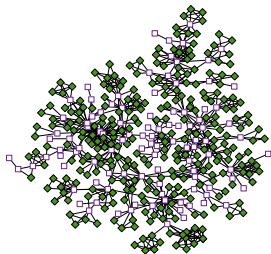
backbones & *skeletons*

introduction to *network analysis* (*ina*)

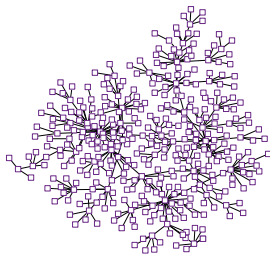
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# skeletons *overview*

- *network backbone* retains *strongest links/information flow*  
sparsification technique that removes as many links as possible
- *network skeleton* retains *micro/meso/macro structure* [CN17]  
simplification technique that retains as many links as possible



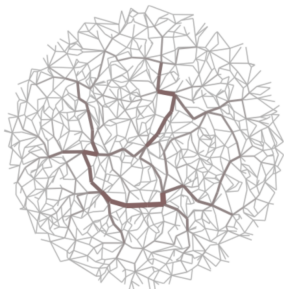
coauthorship network



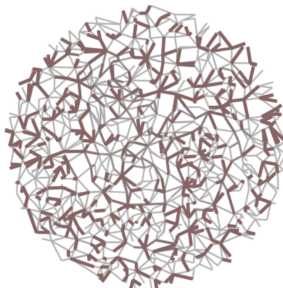
network spanning tree

# skeletons *geodesics*

- *betweenness/saliency skeletons* of *synthetic graph* [GTB12]  
betweenness/saliency  $\sim$  number of geodesics/spanning trees including edge



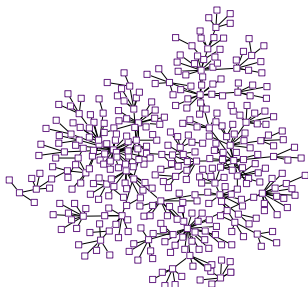
high-betweenness backbone



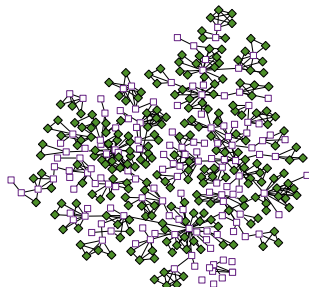
high-saliency skeleton

# skeletons *convexity*

- *convex skeleton* of famous *network scientists* [MŠ18, Šub18]  
largest skeleton where every subgraph includes most geodesics



network spanning tree



convex skeleton  $\sim$  tree with cliques

# skeletons *statistics*

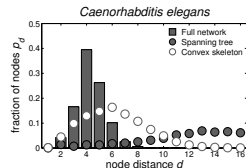
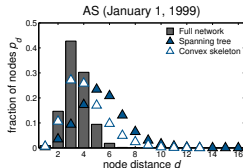
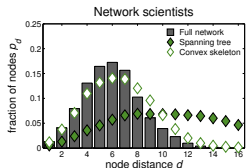
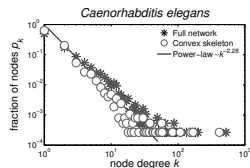
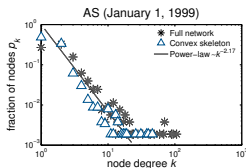
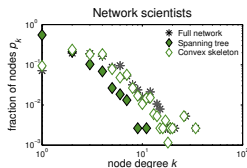
- *large-scale statistics* of *skeletons of networks* [Šub18]  
convex skeletons retain connectivity, clustering and geodesics

	clustering $\langle C \rangle$			geodesics $\langle \sigma \rangle$			convexity $X_s$		
	N	CS	ST	N	CS	ST	N	CS	ST
Jazz musicians	0.62	0.81	0.00	9.71	1.97	1.00	0.12	0.84	1.00
Network scientists	0.74	0.75	0.00	2.66	1.47	1.00	0.85	0.95	1.00
Computer scientists	0.48	0.54	0.00	4.08	1.42	1.00	0.64	0.95	1.00
<i>Plasmodium falciparum</i>	0.02	0.07	0.00	3.71	1.77	1.00	0.43	0.95	1.00
<i>Saccharomyces cerevisiae</i>	0.07	0.10	0.00	2.58	1.19	1.00	0.68	0.88	1.00
<i>Caenorhabditis elegans</i>	0.06	0.12	0.00	6.79	3.03	1.00	0.56	0.85	1.00
AS (January 1, 1998)	0.18	0.21	0.00	3.87	2.32	1.00	0.66	0.91	1.00
AS (January 1, 1999)	0.18	0.27	0.00	3.54	2.05	1.00	0.49	0.95	1.00
AS (January 1, 2000)	0.20	0.25	0.00	4.81	3.07	1.00	0.59	0.90	1.00
Little Rock Lake	0.32	0.69	0.00	22.13	4.32	1.00	0.02	0.82	1.00
Florida Bay (wet)	0.33	0.79	0.00	9.17	1.37	1.00	0.03	0.92	1.00
Florida Bay (dry)	0.33	0.82	0.00	9.37	1.65	1.00	0.03	0.93	1.00

# skeletons *distributions*

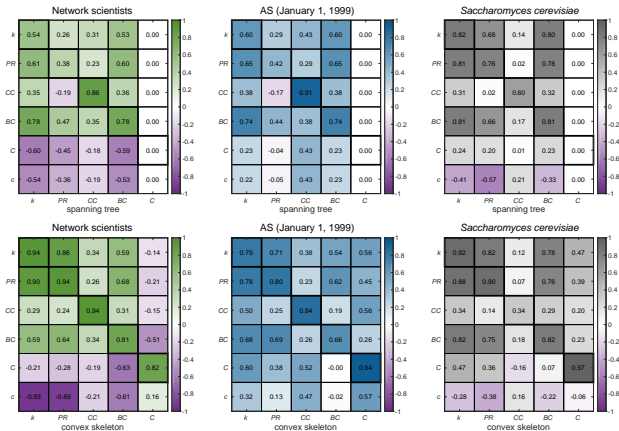
— *node distributions* of *skeletons of networks* [Šub18]

convex skeletons retain node degrees and distances



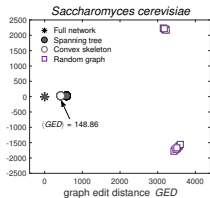
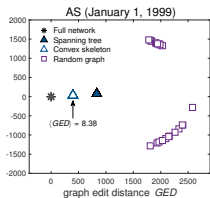
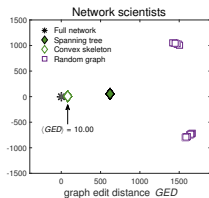
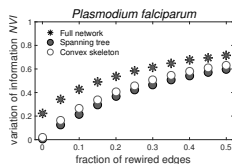
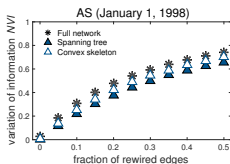
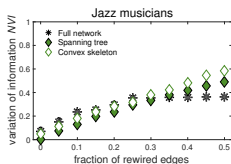
# skeletons *centralities*

- *node centralities* in *skeletons of networks* [Šub18]  
convex skeletons retain node centralities and clustering



# skeletons *communities*

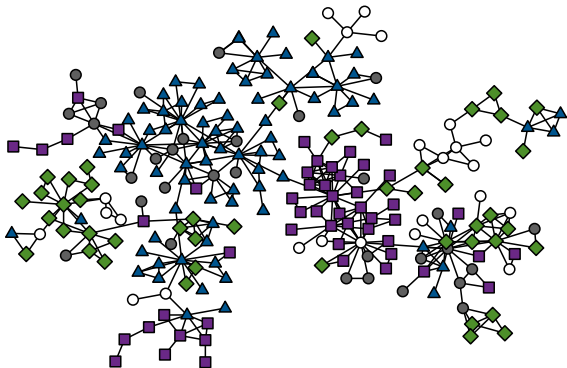
- *communities & robustness of skeletons of networks* [Šub18]  
convex skeletons highlight communities with short edit distance





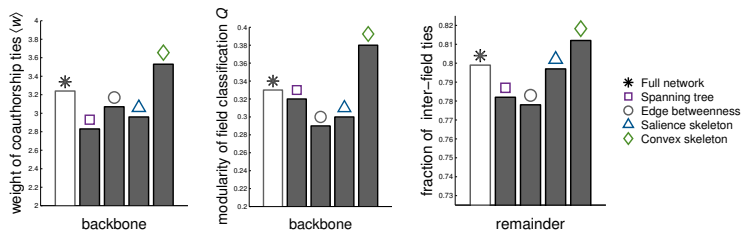
# skeletons *coauthorships*

— *convex skeleton* of SI *computer scientists* [Šub18, ŠFCK19]  
computer theory (◆), information syst. (■), intelligent syst. (▲), programming (○) etc.



# skeletons *properties*

- *properties* of *skeletons of coauthorships* [Šub18, ŠFCK19]  
convex skeletons retain strong intra-field coauthorships



# skeletons *references*



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# skeletons *references*



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