small-world networks

introduction to network analysis (ina)

Lovro Šubelj University of Ljubljana spring 2024/25

small-world *documentary*

connected the power of six degrees

documentary on small-world and scale-free networks

profit down thin - third 's sight after sportantly limit for a second to cover random direct 'n sported spinst or itse difference range and below. The spinst profit of the spinst sector spinst and the spinst spinst or each transmit profits after spinst spinst after second to random profits after spinsters & 0. This was compared by a sector spinster spin sector spin terms and the spinster spinster spin sector spin spin sector spinster spinster spinster spin sector spin spin sector spinster spin sector spin sector spin sector spin spin sector spin sector spin sector spin sector spin sector spin sec- tor spin sector spin sector spin sector spin sector spin sec- tor spin sector spin se	Collective dynamics of 'small-world' networks
dat is assumed to cannot challing wheth? We object optimise of each silication cannot gain the data is a sum of a state of paratic architect frammar (1994). A start gain of the state of the state of paratic start (1994) whether the the state of the sta	Collective dynamics of 'small-world' networks
Conserve process across staticities around, hence process adjust on our her second positive staticities. X. R. Hone and the second static programmer are second process and second staticities. Manufactures and static static process staticities and process static range, the static process static from the poly others in the second static process static from the poly others in the second state.	'amail-world' networks
elabel on out to manual particle statement (\$10), then and it is associated a population, and manual is particle on completely address. Therefore any methods in the statement of the decision range on particle statement (statement of out of out or statement integers in statement (statement of out of out or statement integers in statement (statement of out of out of out or statement integers in statement (statement of out of out of out of out of outputs).	and were moneth
the completely addes The sphere is a state provided for sugger the observ- ring are prove strend (facts and observations repor- lings). Its setting factors prior high tables to solve the	
	to a set of the set of
	Approach (Through on Apple Delaws, The of Mal
ingles the shell alread strates are to a done a their ingle and particle bands strategies constants of early et- angle and particles and having a strategies with derivative and have and have a strategies with derivative and have and have a their particular order. Instant have	Serger and Ser. Support parts and ", and and support and service", such as an extent of the service of the s
Bolis of andres of parts of a set of the set	Note on explore study analysis of indexade that one for feasing this works, product splitter schedule trained to the decision of product splitters for that the feasing one is the high methods the scalar before of the decision of the splitter the scalar people. We are
and frequency in the set	Phonese " post-times a Agendance
	The shared activate of the source frameworkshifts along prices good of the sources funded from a source in confiden- priph of the activate on these to be source and the confiden- tion of the activation of shores to be a source of the limited of the activation of shores to be a final of the activation of shores to be a final of the activation of the shore of the activation of the activation of the shore of the shore of the activation of the shore of the shore of the shore of the activation of the shore of the shore of the shore of the activation of the shore of the shore of the shore of the activation of the shore of the shore of the shore of the activation of the shore of the shore of the shore of the shore of the activation of the shore of the shore of the shore of the shore of the activation of the shore of the shore of the shore of the shore of the activation of the shore of the shore of the shore of the shore of the activation of the shore of the shore of the shore of the shore of the activation of the shore of the shore of the shore of the shore of the activation of the shore of the shore of the shore of the shore of the activation of the shore of the shore of the shore of the shore of the activation of the shore of
	and a sumb and about the transition form. To exception former signal and an about infection and the following studies conting provides the fol- bation of prime with a context and only of the signal control of the studies of the studies and the signal studies in form the problem of provides () for a control of the following the studies of the studies of the studies in form the problem of provides () for a control of the test and provides of the studies of the studies of the test and the studies of the studies of the studies of the test and the studies of the studies of the studies of the test and the studies of the studies of the studies of the test and the studies of the studies of the studies of the test and the studies of the studies of the studies of the test and the studies of the studies of the studies of the test and the studies of the studies of the studies of the test and the studies of the studies of the studies of the test and the studies of the studies of the studies of the test and test and test and the studies of the studies of the test and test and test and test and test and test and test and test test and test
	which this is based. The specify the characteristic properties of these payles is developed in part barged in (2) and characteristic coefficient obtained only it happens that is (2) subseque for the payles to be based for a solution of the payle is pathways with the baracteristic characteristic of a logical barged solution.
	property. The indexed of second is as here more to only proceedings of the originary for the pro- tinger of forwards descended. Specifically us with the constant of these parameters for our paids with a constant of the specifical to the constant of the specifical second second second output the specifical second second second second second by the specifical second
	In a highly channel, heigh world when it gives thereity where the market indexed it if a rest to a party channel world when it gives with paperbiased with a three to market the iteration of the second second second term is not reached of white and it is the second second second second second second from the second second second second second second in the second second second second second second second in the second second second second second second second in the second secon
	are which high is denoted a standard in barry of $(0,1)$. The Barry and a standard method would be the transmission of a standard by the standard test of a first heigh standard day are standard barry. For standard test of the heigh standard field $\frac{1}{2}$, the standard test of the heigh standard test of the standard barry of the height standard test standard barry of the height standard barry of the standard barry of the standard barry of the height standard barry of the standard barry of the standard barry of the height standard barry of the standard barry of the standard barry of the height standard barry of the standa
and the set of the set	And the last of th

[WS98]



[BA99]

etters to nature	
	a liste C.I. Roby at the optic base has been been up a
international as posses, and a contacted provide spin-	1 Chang 1 F at the A free structure of the change of many prime, product a first field states in the structure of the
the bounded is an ended with other than of an ending it.	A CONTRACT OF THE OWNER AND A DESCRIPTION OF
a state state in the state and a side of the	These Schools Revised and American Sciences and American Sciences
	A contract of the location of the state in the location
	a side in this way in the data product, chapter, but anylo (the
a substant they untilling it and is done effort	A close A children i de parte a ser de contrate des
reneral tech for the orders of theory days 5 first exp. or	CONTRACTOR PROPERTY AND ADDRESS OF THE OWNER OWNER OF THE OWNER
	a set of sectors of the local data and the sector of the s
he has the date parties is a 17had students to figure 1	(Actual)
tenders forband) had the problem complex and match.	to the set of the set
to be set if and suggest if yets of the back-back map	a Descript 1 (and the description in a chapter into the second state in the second state of the second
taking a 7b they by out statestic day h	A Report Cold Andrew States and American American American
a school of a feet school of a day for the feet build	a literative of a literative base of the second sec
	and an address of the state of the state of the
	A appropriate the second
	And the second s
A 14 14 14	to the state of the local of the second
	In the second se
	A new rest of the lot of the balance of the layers of specific states
the plane graduate the apply the sales to a backdonous and	A loss that is the set of the set
	Adapted of production in the local state of the
	the state of the state of the local state.
hadd and be enforted to the Bullachine to be seen from	to be a subscript to be a subscript here in the part of the state state, but a
a subject planting higher despired, moder in plane with the	Caller 1 - Caller C. Second day of and planet place.
	to be a first of the second seco
It is neared in Annual generation increase of place labor.	A surface loss of the second s
operations is its lagrang. In plane is an instant out	Comparing a Logencia model double diseases 11
	and the second address in the second se
	of an article of the barrenese
and a second second second second second	
	Error and attack televance
and characterized and	
	et complex networks
	The Real Process lines and that was been
	Approach of the Without Standing Street, of the loss
	Nacionalist come diglic a seguring dagoe of chicken
income of poster the matter control (but is, how comp).	speet cross for cample whitely made organize pro-
for only of 1 with choosing growth depice security	post of spale date date pleasand a
	the solution of the mobility mobility reduced in
New Selected as a reason theory of a particular sector.	constation actually high completing it production.
to additionable decharation relation of the proster of	
disease from the second s	diffs of its televish. The public of here and when complete
hand Case, or policity has	come is the excitence is the selector error of the law.
	hand all difficilly its spinse' composite Review dimen-

[AJB00]

small-world phenomenon



 $\begin{array}{l} -- \langle d \rangle = 4.74 \mbox{ for } \textit{Facebook } \mbox{ friendships } [BBR^+12] \mbox{ while } \frac{\ln n}{\ln \langle k \rangle} = 3.98 \\ -- \langle C \rangle = 0.61 \mbox{ for } \textit{Facebook } \mbox{ social circles } [NL12] \mbox{ while } \rho < 10^{-6} \end{array}$

small-world *model*

— G(n, k, p) small-world model [WS98]

- randomly rewire pnk/2 links of regular lattice
- conceptually interesting but practically inapplicable
 - for some *p* small-world with $\langle d \rangle \simeq \frac{\ln n}{\ln \langle k \rangle}$ and $\langle C \rangle \gg 0$
 - for p = 1 random graph with $\langle d \rangle \simeq \frac{\ln n}{\ln \langle k \rangle}$

- for
$$p = 0$$
 regular lattice with $C = \frac{3(k-2)}{4(k-1)}$



see small-world model NetLogo demo

small-world networks

- clustering coefficient $\langle C \rangle$ in real small-world networks

- average distance $\langle d \rangle$ in real small-world networks

network	п	$\langle C \rangle$	$\gg rac{\langle k angle}{n-1}$	$\langle d \rangle$	$\approx \frac{\ln n}{\ln \langle k \rangle}$
southern women [DGG41]	32	0.000	0.179	2.31	2.02
karate club [Zac77]	34	0.571	0.139	2.41	2.31
American football [GN02]	115	0.403	0.094	2.51	2.00
Java dependencies [ŠB11]	1368	0.497	0.012	2.21	2.59
Facebook circles [ML12]	4039	0.606	0.011	3.69	2.20
physics collaboration [New01]	36 458	0.657	0.000	5.50	4.68
Enron e-mails [LLDM09]	36 692	0.497	0.001	3.39	3.51
Internet map [HJJ ⁺ 03]	75 885	0.160	0.000	5.83	5.01
actors collaboration [BA99]	382 219	0.780	0.000	pprox 3.6	2.94
physics citation [ŠFB14]	438 943	0.227	0.000	pprox 5.0	4.23
patent citation [HJT01]	3 774 768	0.076	0.000	pprox 8.1	6.98
Facebook snowball [Fer12]	8 217 272	0.019	0.000	pprox 6.8	14.23

small-world experiments

- 6 degrees of separation in letter passing as $\langle d \rangle = 6.2$ [Mil67]
- 4/7 degrees of separation in e-mail communication [DMW03]
- 4 degrees of separation on Facebook as $\langle d \rangle = 4.74$ [BBR⁺12]



— the strength (weakness) of weak (strong) ties [Gra73]

small-world *navigation*

does existence of short paths imply navigable small-world by decentralized search? [KIe00]

small-world *history*





Reka Albert, Hawoong Jeong, and Albert Laszlo Barabasi. Error and attack tolerance of complex networks. *Nature*, 406(6794):378–382, 2000.



A.-L. Barabási and R. Albert.

Emergence of scaling in random networks. Science, 286(5439):509-512, 1999.



A.-L. Barabási.

Network Science. Cambridge University Press, Cambridge, 2016.



Lars Backstrom, Paolo Boldi, Marco Rosa, Johan Ugander, and Sebastiano Vigna.

Four degrees of separation.

In Proceedings of the ACM International Conference on Web Science, pages 45–54, Evanston, IL, USA, 2012.



A. Davis, B. B. Gardner, and M. R. Gardner.

Deep South. Chicago University Press, Chicago, 1941.



Peter Sheridan Dodds, Roby Muhamad, and Duncan J. Watts. An experimental study of search in clobal social networks. *Science*, 301(5634):827–829, 2003.



Wouter de Nooy, Andrej Mrvar, and Vladimir Batagelj. Exploratory Social Network Analysis with Pajek: Expanded and Revised Second Edition. Cambridge University Press, Cambridge, 2011.



David Easley and Jon Kleinberg.

Networks, Crowds, and Markets: Reasoning About a Highly Connected World. Cambridge University Press, Cambridge, 2010.



Ernesto Estrada and Philip A. Knight. A First Course in Network Theory. Oxford University Press, 2015.



P. Erdős and A. Rényi.

On random graphs I. Publ. Math. Debrecen, 6:290-297, 1959.



Stefano Ferretti.

On the degree distribution of faulty peer-to-peer overlays. ICST Transactions on Complex Systems, 2012.



M. Girvan and M. E. J Newman.

Community structure in social and biological networks. P. Natl. Acad. Sci. USA, 99(12):7821-7826, 2002.



Mark S. Granovetter.

The strength of weak ties. Am. J. Sociol., 78(6):1360-1380, 1973.



M Hoerdt, M Jaeger, A James, D Magoni, J Maillard, D Malka, and P Merindol. Internet {IP}v4 overlay map produced by network cartographer (nec), 2003.



B. H. Hall, A. B. Jaffe, and M. Tratjenberg.

The NBER patent citation data file: Lessons, insights and methodological tools. Technical report, National Bureau of Economic Research, 2001.



Jon M. Kleinberg.

Navigation in a small world. Nature, 406(6798):845, 2000.



Jure Leskovec, Kevin J Lang, Anirban Dasgupta, and Michael W Mahoney.

Community structure in large networks: Natural cluster sizes and the absence of large well-defined clusters. Internet Math., 6(1):29–123, 2009.



Stanley Milgram.

The small world problem. Psychol. Today, 1(1):60-67, 1967.



Seth A. Myers and Jure Leskovec.

Clash of the contagions: Cooperation and competition in information diffusion. In Proceedings of the IEEE International Conference on Data Mining, 2012.



M. E. J. Newman.

The structure of scientific collaboration networks. *P. Natl. Acad. Sci. USA*, 98(2):404–409, 2001.

Mark E. J. Newman.

Networks. Oxford University Press, Oxford, 2nd edition, 2018.



Azree Nazri and Pietro Lio.

Investigating meta-approaches for reconstructing gene networks in a mammalian cellular context. PLoS ONE, 7(1):e28713, 2012.



Lovro Šubelj and Marko Bajec.

Community structure of complex software systems: Analysis and applications. *Physica A*, 390(16):2968–2975, 2011.



Lovro Šubelj, Dalibor Fiala, and Marko Bajec.

Network-based statistical comparison of citation topology of bibliographic databases. Sci. Rep., 4:6496, 2014.



D. J. Watts and S. H. Strogatz.

Collective dynamics of 'small-world' networks. *Nature*, 393(6684):440–442, 1998.



Wayne W. Zachary.

An information flow model for conflict and fission in small groups. J. Anthropol. Res., 33(4):452–473, 1977.