war pact network model:
generative model of networks that shrink

Lovro Šubelj
University of Ljubljana
Faculty of Computer and Information Science

joint work with

Luka Naglić
University of Zagreb
Faculty of Science

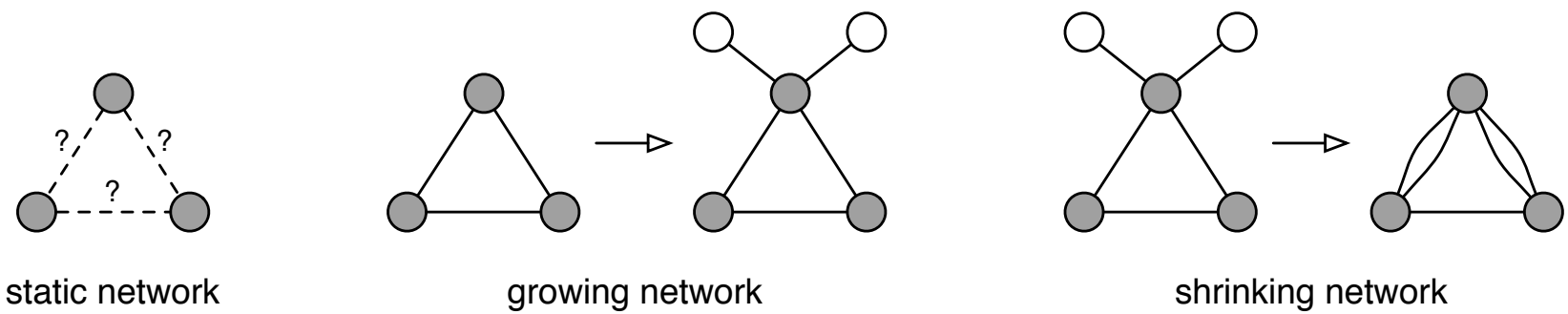
EUSN '19

network **models**

(**soa**) network **models** as **baseline**, **explanation** & **generation**

(**existing**) majority for **static** or **growing networks** [**ER59**, **Pri76**]

(**missing**) **generative models** of **shrinking networks** [**KNB08**]



[**ER59**] Erdős & Rényi (1959) On random graphs I. *Publ. Math. Debrecen* **6**, 290-297.

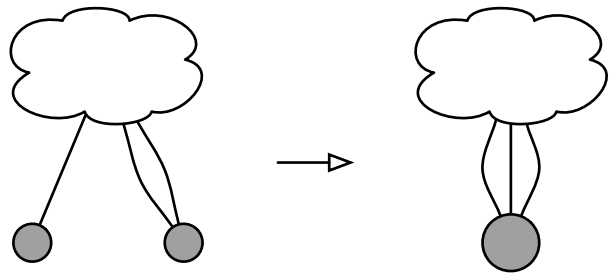
[**Pri76**] Price (1976) A general theory of bibliometric and other cumulative... *J. Am. Soc. Inf. Sci.* **27**(5), 292-306.

[**KNB08**] Kejžar et al. (2008) Probabilistic inductive classes of graphs. *J. Math. Sociol.* **32**(2), 85-109.

shrinking models

(intuition) entities/nodes often merge in real world/network

(which) merged nodes/entities are random, hubs, isolates etc.



(wars) nations/alliances form pact or one occupies other

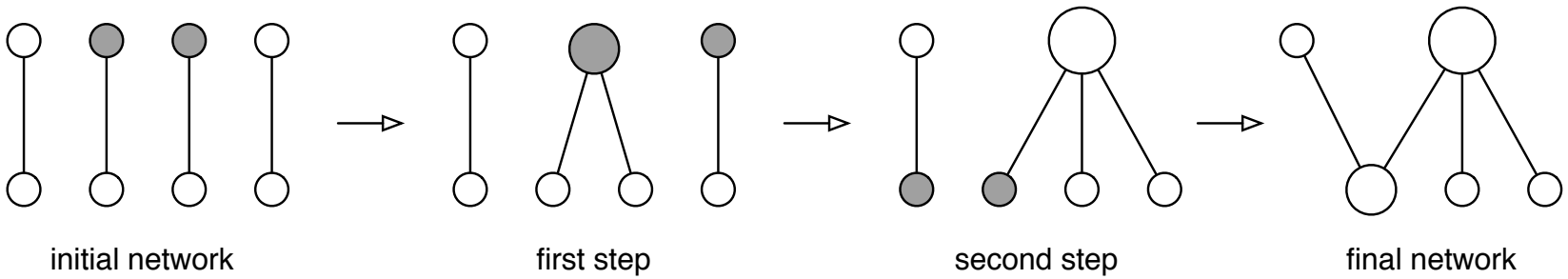
(trade) countries form alliance or companies after merger

(Bitcoin) cryptocurrency addresses owned by same user

(Internet) autonomous systems merge their traffic

war pact model

(model) shrinking network with n nodes & m edges



(initialize) create perfect matching on $2m$ nodes

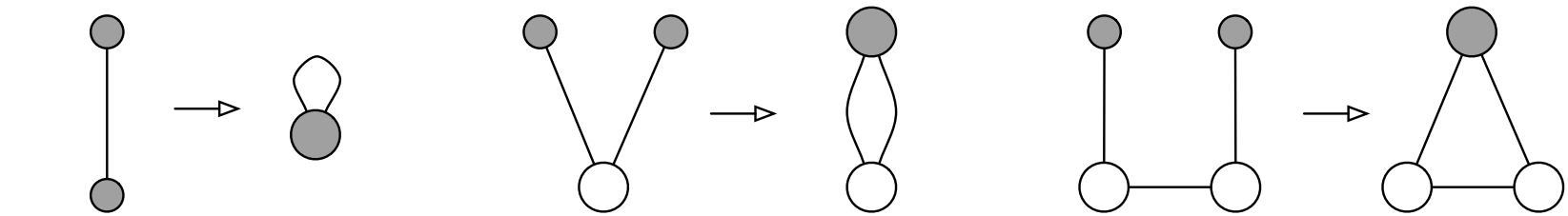
(select) select nodes at random, preferentially etc.

(shrink) merge nodes by rewiring their edges

(loop) continue until network has n nodes

model **details**

(**shrink**) **merging nodes** at **distance** *d* creates ***d*-cycle**



(**model**) war pact is **parameter-free** except *n* **nodes** & *m* **edges**

(**initialize**) create **perfect matching**, **random graph** or **tree** ○

(**select**) select nodes at **random**, by **degree** or **degree**$^{-1}$ ●

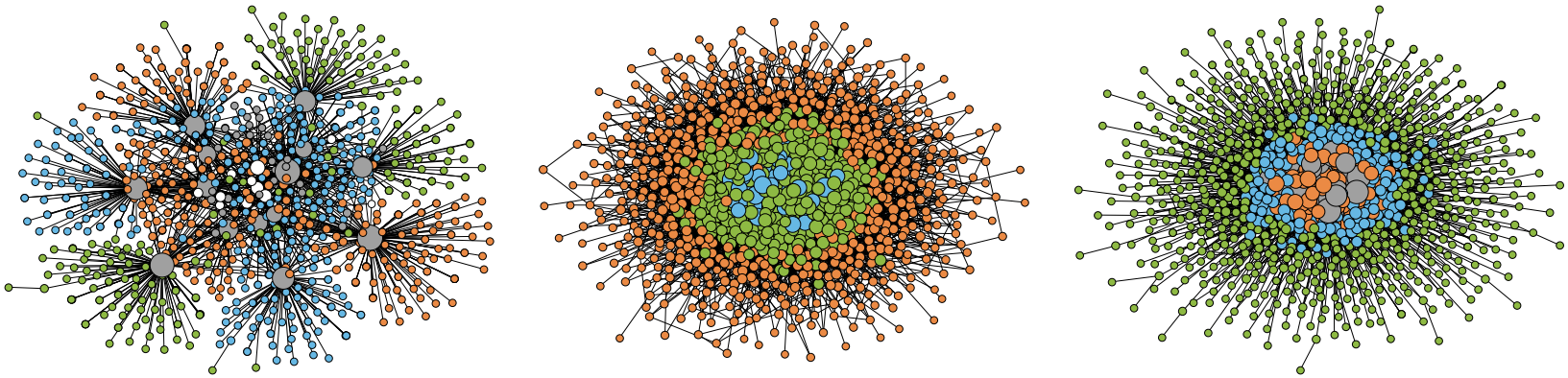
model **pseudocode**

input nodes n & edges m
output graph G

1: $H \leftarrow$ **empty map** ▷ map of nodes' hashes
2: $G \leftarrow$ **empty graph** ▷ empty war pact graph
3: **for** $i \in [1, m]$ **do**
4: $H(i) \leftarrow i$ & $H(m+i) \leftarrow m+i$ ▷ map nodes to hashes
5: **add nodes** $H(i)$ & $H(m+i)$ to G ▷ add nodes to graph
6: **add edge** $\{H(i), H(m+i)\}$ to G ▷ add edges to graph
7: **while** G has $> n$ nodes **do**
8: $h \leftarrow$ **random(H)** ▷ select random node
9: $i \leftarrow$ **random([1, 2m])** ▷ select node by degree
10: **if** $h \neq H(i)$ & edge $\{h, H(i)\} \notin G$ **then**
11: **merge nodes** h **&** $H(i)$ in G ▷ merge selected nodes
12: $H(i) \leftarrow h$ ▷ unify nodes' hashes
13: **return** G

model **networks**

(**layout**) **node selection** impacts (**modular**) **structure** [**Pei18**]



(**left**) both nodes are selected **by degree**

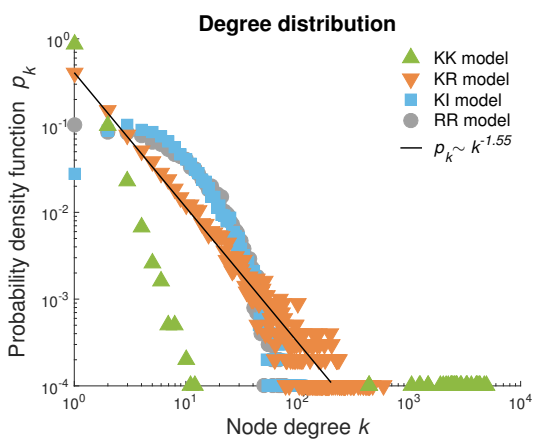
(**middle**) nodes selected **by degree** & **degree**$^{-1}$

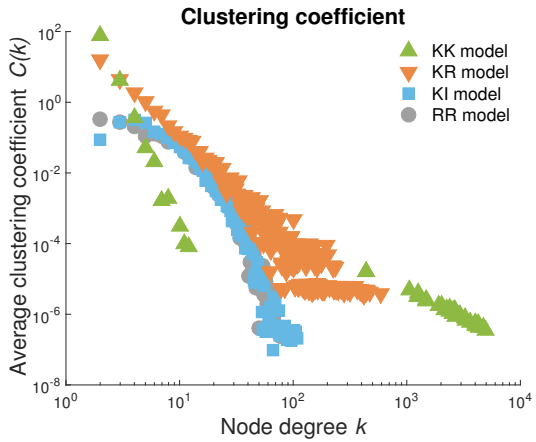
(**right**) nodes selected **by degree** & **at random**

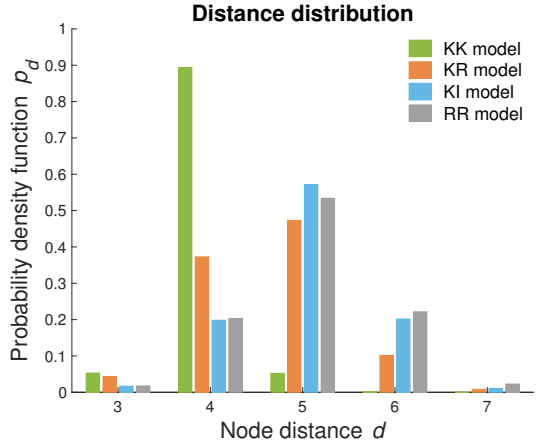
[**Pei18**] Peixoto (2018) Bayesian stochastic blockmodeling. e-print *arXiv:1705.10225v7*, 1-44.

model selection

(structure) node selection impacts scale-free/small-world







(**KK model**) both are nodes selected **by degree**

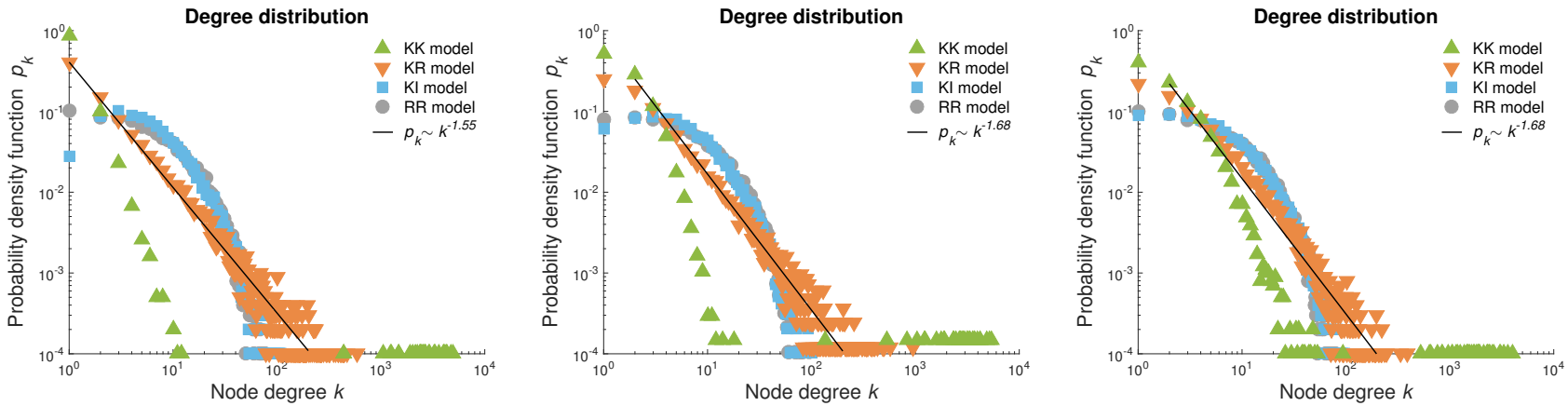
(**KR model**) nodes selected **by degree** & **at random**

(**KI model**) nodes selected **by degree** & **degree**$^{-1}$

(**RR model**) both nodes are selected **at random**

model **initialization**

(**structure**) **model initialization has no** apparent **impact**



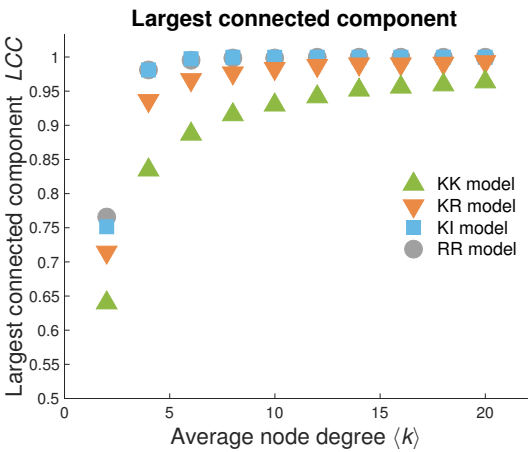
(**left**) networks initialized by **perfect matching**

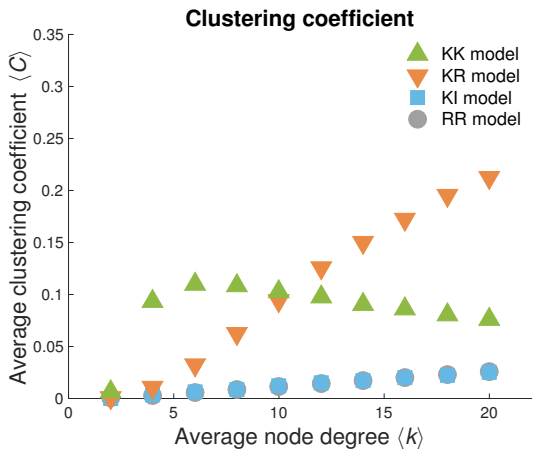
(**middle**) networks initialized by **random graph**

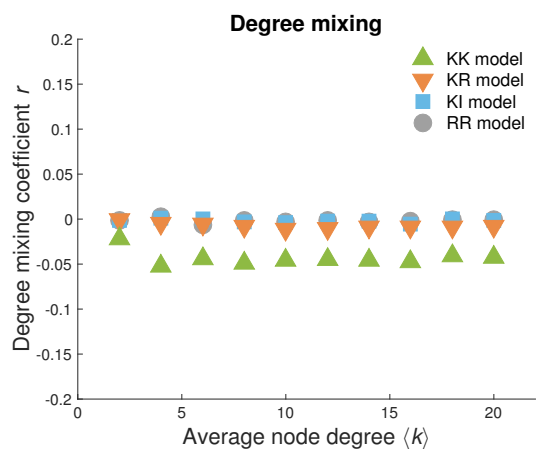
(**right**) networks initialized by **random tree**

model evolution

(structure) model evolution when increasing node degree $\langle k \rangle$







(left) emergence of giant component LCC when increasing $\langle k \rangle$

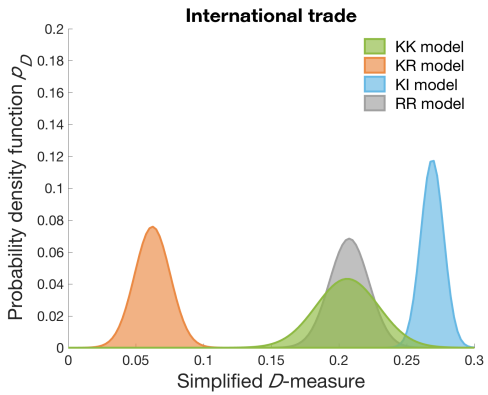
(middle) increasing node clustering $\langle C \rangle$ when increasing $\langle k \rangle$

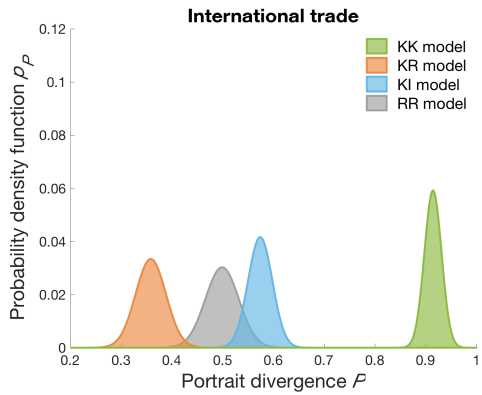
(right) "fixed" degree mixing r when changing $\langle k \rangle$

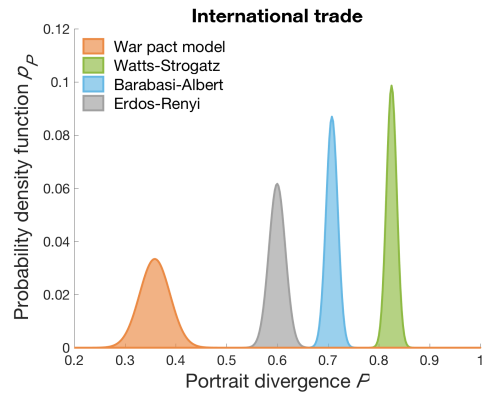
model **comparison**

(**network**) **international trade** (i.e. food import & export)

(**models**) **war pact** $\gg$ **small-world**, **scale-free** & **random** graphs







(**left**) simplified D-**measure** [**SCDPMR17**]

(**right**) **portrait divergence** P [**BB19**]

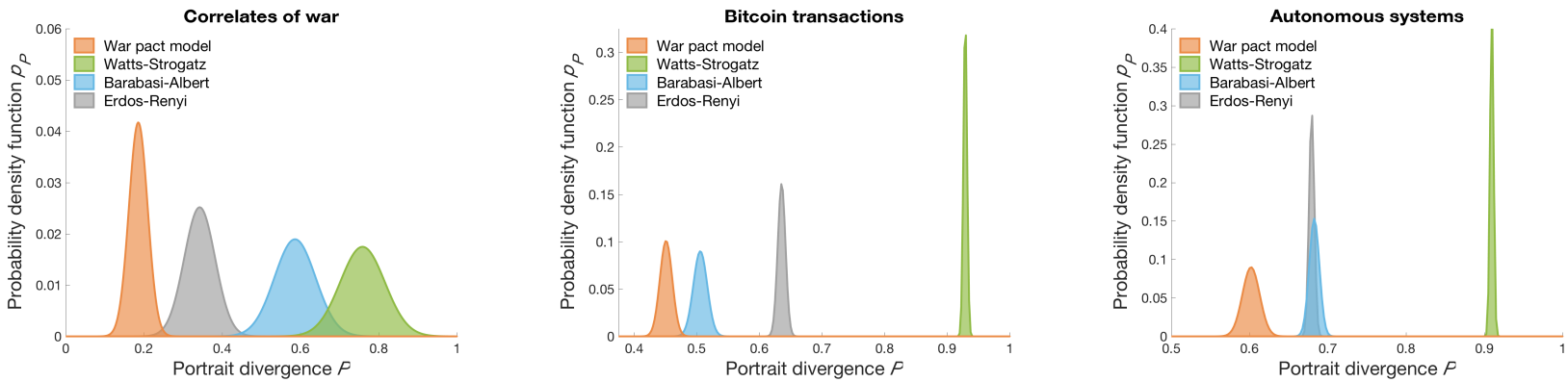
[**SCDPMR17**] Schieber et al. (2017) Quantification of network structural dissimilarities. *Nat. Commun.* **8**, 13928.

[**BB19**] Bagrow & Bollt (2019) An information-theoretic, all-scales approach to comparing. . . *Appl. Netw. Sci.* **4**, 45.

model **validation**

(**networks**) **national wars**, **Bitcoin** transactions & **Internet** map

(**models**) **war pact** $\gg$ **small-world**, **scale-free** & **random** graphs



(**measure**) **portrait divergence** P [**BB19**]

[**BB19**] Bagrow & Bollt (2019) An information-theoretic, all-scales approach to comparing. . . *Appl. Netw. Sci.* **4**, 45.

model **structure**

(**size**) model **reproduces nodes** n & **edges** m **by design**

(**connectivity**) model **well reproduces giant component** LCC

(**distance**) model **well reproduces distance** $\langle d \rangle$ & **diameter** d_{max}

	n	m	$\langle k \rangle$	LCC	$\langle C \rangle$	$\langle d \rangle$	d_{max}
Correlates of war	41	54	2.63	87.8%	**0.28**	2.58	8
	41	54	2.63	90.2%	**0.06**	2.64	7
International trade	130	3 730	57.38	100.0%	0.50	2.24	5
	130	3 730	57.38	100.0%	0.53	2.17	5
Bitcoin transactions	1 288	6 236	9.68	98.8%	**0.33**	2.83	9
	1 288	6 236	9.68	98.0%	**0.13**	3.08	7
Autonomous systems	3 213	11 248	7.00	100.0%	**0.18**	3.77	9
	3 213	11 248	7.00	98.3%	**0.03**	3.62	9

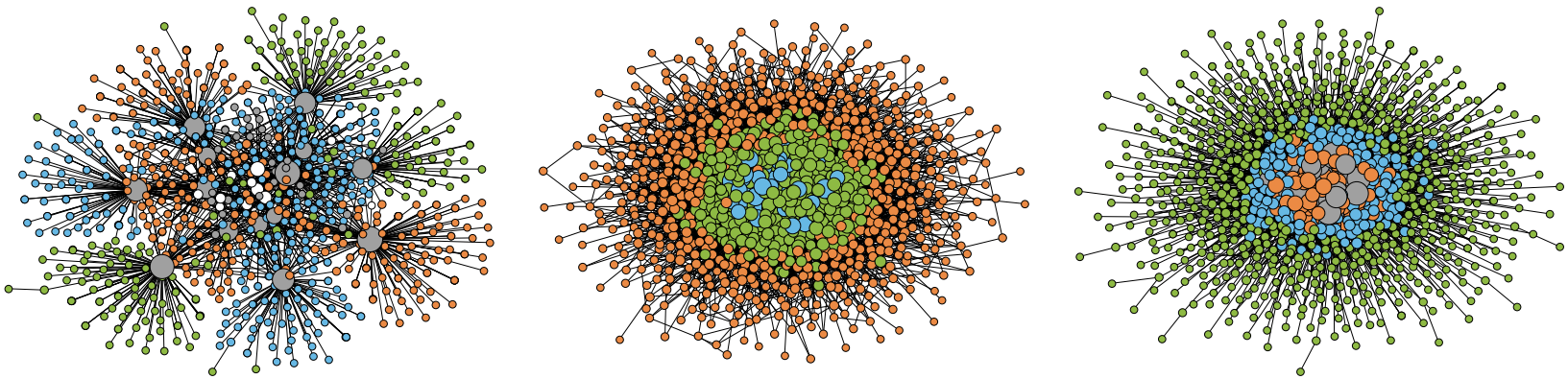
(**clustering**) model often **underestimates node clustering** $\langle C \rangle$

model **conclusions**

(**novel**) **simple model** of **networks that shrink**

(**others**) **in contrast** to classic **static** & **growing models**

(**networks**) model **well reproduces structure** except clustering



(**question**) **growing** or **shrinking models** more "**reasonable**"?

(**future**) **combined model**, other networks & analytical results

thank you!

arXiv:1909.00745v1

Naglić & Šubelj (2019) War pact model of shrinking networks. *PLoS ONE*, under review.

Lovro Šubelj
University of Ljubljana
lovro.subelj@fri.uni-lj.si
http://lovro.lpt.fri.uni-lj.si

joint work with

Luka Naglić
University of Zagreb
lu.naglic@gmail.com
http://www.pmf.unizg.hr