spanning trees that preserve network distances

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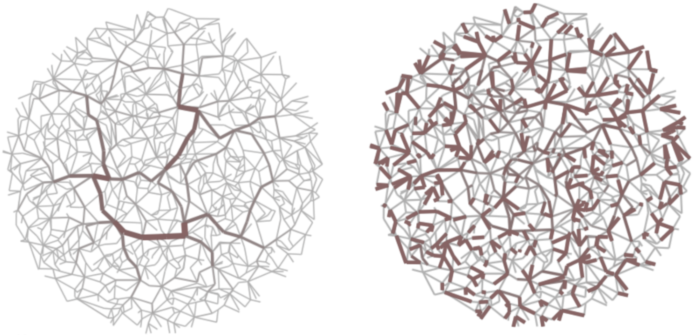
adapted from Networks '21

motivation

network abstraction with backbones and skeletons

(left) high-betweenness backbone and (right) high-salience skeleton (Grady et al., 2012)



spanning trees

network abstraction with **spanning trees**

consider connected unweighted network on n nodes

spanning tree is **connected with n nodes** and $n-1$ edges

trees lack clustering $\langle C \rangle = 0$ in contrast to convex skeletons (Šubelj, 2018)

are spanning trees also **small-world and scale-free?**

$\langle d \rangle \sim \log n$ in small-world networks and $p_k \sim k^{-\gamma}$ in scale-free networks

in random trees almost surely $\langle d \rangle \sim \sqrt{n}$ (Reńyi and Szekeres, 1967)

algorithms

Kruskal's algorithm

1. start with forest of trees each consisting of single node
2. merge trees until only one remains (using minimum edges)

Prim's algorithm

1. start with single tree consisting of (random) seed node
2. add one new neighbor of node at each step (using minimum edge)

breadth-first search

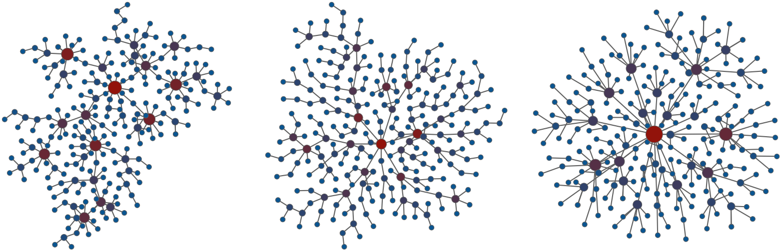
1. start with single tree consisting of (random) seed node
2. add all new neighbors of node at each step (in breadth-first order)

other algorithms

depth-first search, Sollin's and Boruvka's algorithms etc.

examples of spanning trees of random graph

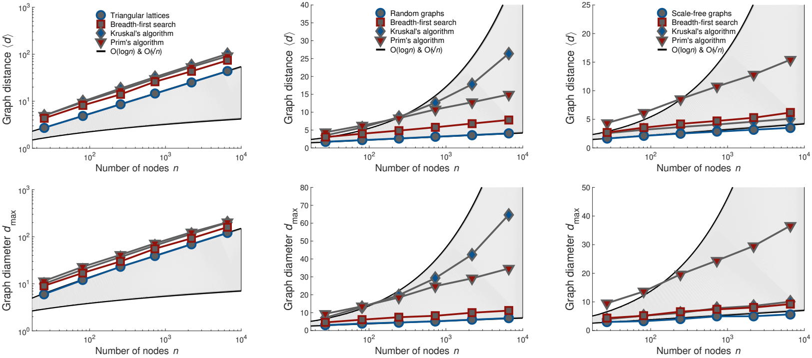
(left) Kruskal's algorithm, (middle) Prim's algorithm and (right) breadth-first search



distance $\langle d \rangle$ and diameter d_{max}

only **BFS retains** scaling of $\langle d \rangle$ **and** d_{max} in synthetic graphs

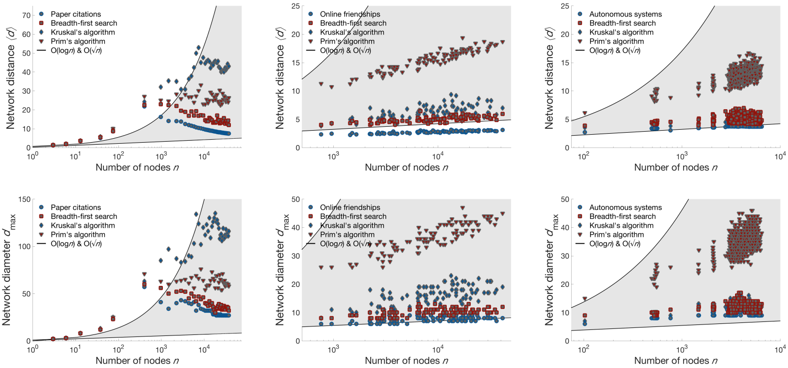
$\langle d \rangle \sim \sqrt{n}$ in lattices, $\langle d \rangle \sim \log n$ in random graphs and $\langle d \rangle \sim \frac{\log n}{\log \log n}$ in scale-free graphs



distance $\langle d \rangle$ and diameter d_{max}

only **BFS retains** scaling of $\langle d \rangle$ **and** d_{max} in real networks

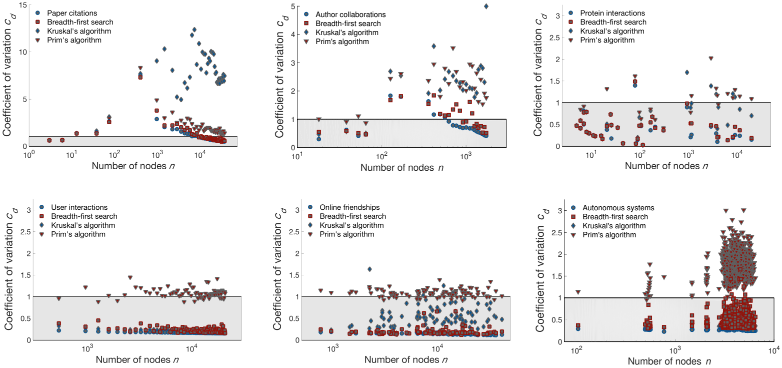
$\langle d \rangle \sim \log n$ in small-world networks and $\langle d \rangle \sim \log \log n$ in ultra small-world networks



distance distribution p_d

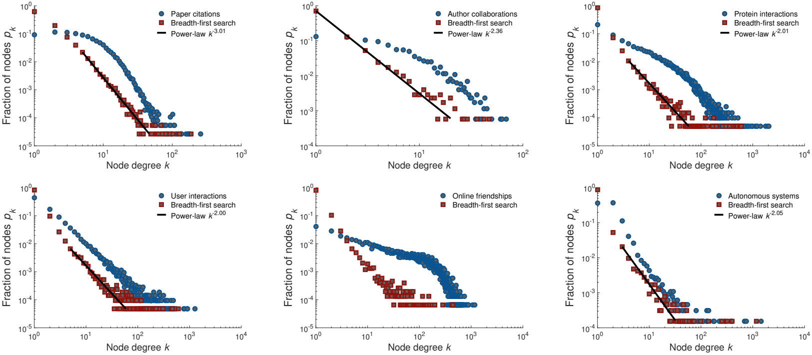
only BFS retains low-variance p_d in real networks

coefficient of variation $c_d = \frac{\sigma_d}{\langle d \rangle} < 1$ in (ultra) small-world networks



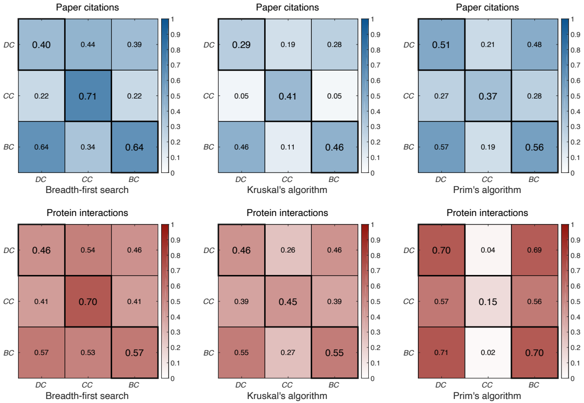
degree distribution p_k

only BFS power-law $p_k \sim k^{-\gamma}$ in most cases (Clauset et al., 2009)



BFS best retains closeness centrality in real networks

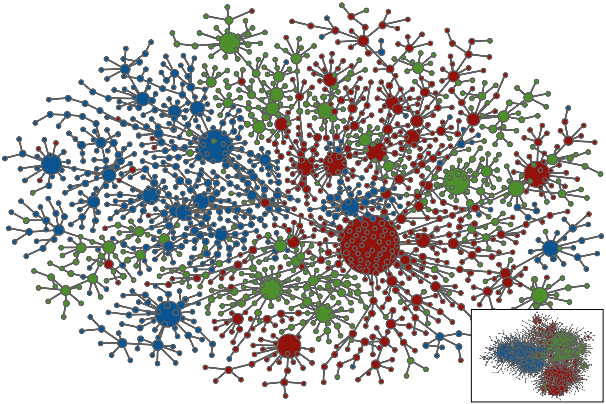
correlations between node degree (*DC*), closeness centrality (*CC*) and betweenness centrality (*BC*)



network visualization

BFS spanning tree of author collaborations in Slovenia

natural sciences (red), engineering (green), medical sciences (blue) and other



conclusions

spanning trees small-world $\langle d \rangle \sim \log n$ and scale-free $p_k \sim k^{-\gamma}$

trees lack clustering $\langle C \rangle = 0$ in contrast to convex skeletons (Šubelj, 2018)

use breadth-first search for unweighted networks!

use Prim's or Kruskal's algorithm only for weighted networks

are breadth-first search trees actually balanced trees?

balanced tree data structure ensures $d_{max} \sim \log n$ by definition

breadth-first search trees are indeed most compact and well-balanced!

thank you!

Šubelj (2025) Computing well-balanced spanning trees of unweighted networks. *Algorithms* **18**(12), 760.

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