

war pact network **model**:
generative model of **networks that shrink**

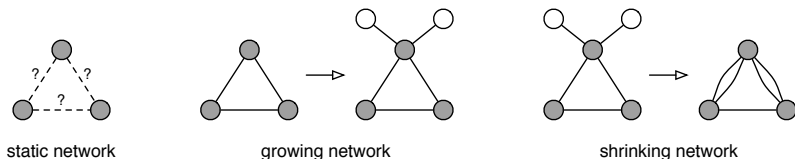
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network models

(**soa**) network **models** as **baseline**, **explanation** & **generation**
(**existing**) majority for **static** or **growing networks** [**ER59**, **Pri76**]
(**missing**) **generative models** of **shrinking networks** [**KNB08**]



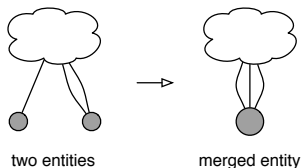
[**ER59**] Erdős & Rényi (1959) On random graphs I. *Publ. Math. Debrecen* **6**, 290-297.

[**Pri76**] Price (1976) A general theory of bibliometric and other cumulative. . . *J. Am. Soc. Inf. Sci.* **27**(5), 292-306.

[**KNB08**] Kejžar et al. (2008) Probabilistic inductive classes of graphs. *J. Math. Sociol.* **32**(2), 85-109.

shrinking models

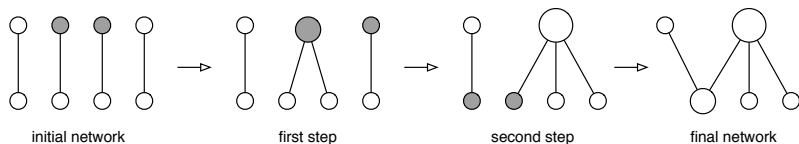
(intuition) **entities/nodes** often **merge** in **real world/network**
(which) **merged nodes/entities** are **random, hubs, isolates** etc.



(wars) **nations/alliances** **form pact** or one occupies other
(trade) **countries** **form alliance** or **companies** **after merger**
(Bitcoin) **cryptocurrency** **addresses** **owned** by same user
(Internet) **autonomous systems** **merge** their traffic

war pact model

(model) **shrinking network** with n nodes & m edges



(initialize) create **perfect matching** on $2m$ nodes

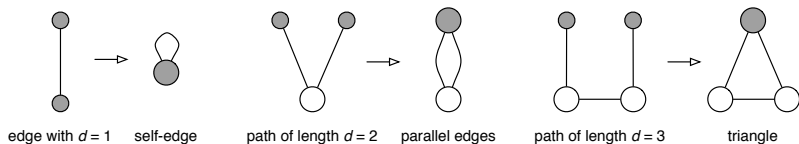
(select) **select nodes** at random, preferentially etc.

(shrink) **merge nodes** by rewiring their edges

(loop) **continue** until network has n nodes

model details

(shrink) merging nodes at distance d creates d -cycle



(model) war pact is **parameter-free** except n nodes & m edges

(initialize) create **perfect matching**, **random graph** or **tree**

(select) select nodes at **random**, by **degree** or **degree⁻¹**

model **pseudocode**

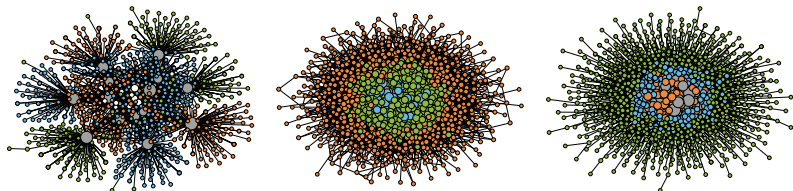
input nodes n & edges m

output graph G

- 1: $H \leftarrow$ **empty map** ▷ map of nodes' hashes
- 2: $G \leftarrow$ **empty graph** ▷ empty war pact graph
- 3: **for** $i \in [1, m]$ **do**
- 4: $H(i) \leftarrow i$ & $H(m+i) \leftarrow m+i$ ▷ map nodes to hashes
- 5: **add nodes** $H(i)$ & $H(m+i)$ to G ▷ add nodes to graph
- 6: **add edge** $\{H(i), H(m+i)\}$ to G ▷ add edges to graph
- 7: **while** G has $> n$ nodes **do**
- 8: $h \leftarrow$ **random**(H) ▷ select random node
- 9: $i \leftarrow$ **random**($[1, 2m]$) ▷ select node by degree
- 10: **if** $h \neq H(i)$ & edge $\{h, H(i)\} \notin G$ **then**
- 11: **merge nodes** h & $H(i)$ in G ▷ merge selected nodes
- 12: $H(i) \leftarrow h$ ▷ unify nodes' hashes
- 13: **return** G

model **networks**

(**layout**) **node selection** impacts (**modular**) **structure** [Pei18]



(**left**) both nodes are selected **by degree**

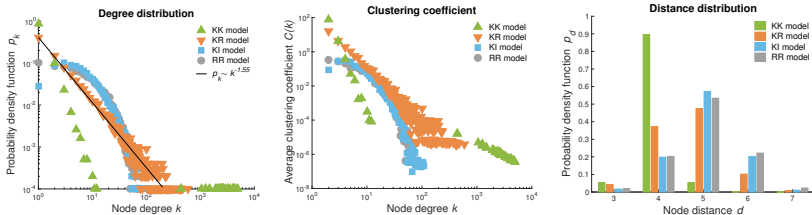
(**middle**) nodes selected **by degree & degree⁻¹**

(**right**) nodes selected **by degree & at random**

[Pei18] Peixoto (2018) Bayesian stochastic blockmodeling. e-print *arXiv:1705.10225v7*, 1-44.

model selection

(structure) node selection impacts scale-free/small-world



(KK model) both are nodes selected by degree

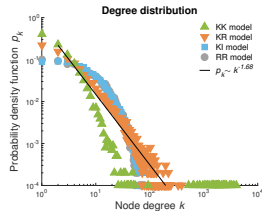
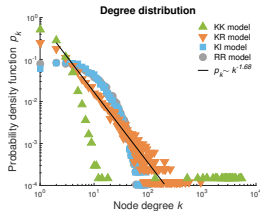
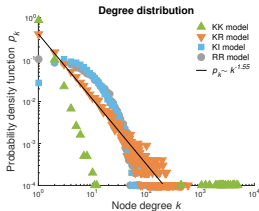
(KR model) nodes selected by degree & at random

(KI model) nodes selected by degree & degree⁻¹

(RR model) both nodes are selected at random

model initialization

(structure) model initialization has no apparent impact



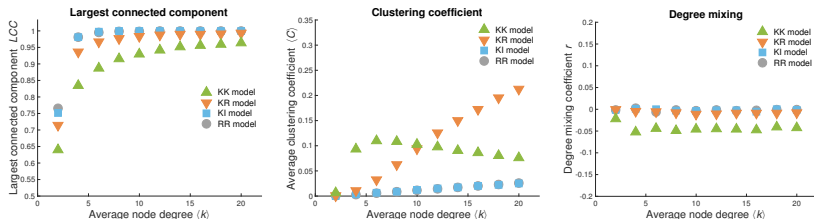
(left) networks initialized by **perfect matching**

(middle) networks initialized by **random graph**

(right) networks initialized by **random tree**

model evolution

(structure) model evolution when increasing node degree $\langle k \rangle$



(left) emergence of giant component LCC when increasing $\langle k \rangle$

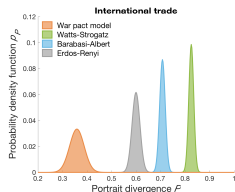
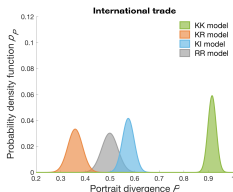
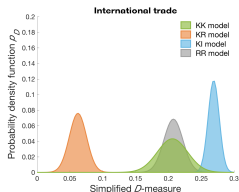
(middle) increasing node clustering $\langle C \rangle$ when increasing $\langle k \rangle$

(right) “fixed” degree mixing r when changing $\langle k \rangle$

model comparison

(network) international trade (i.e. food import & export)

(models) war pact \gg small-world, scale-free & random graphs



(left) simplified D -measure [SCDPMR17]

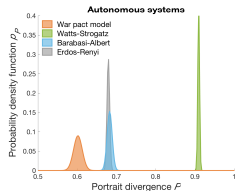
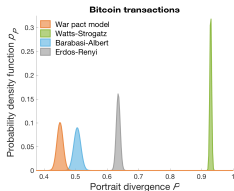
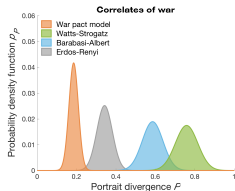
(right) portrait divergence P [BB19]

[SCDPMR17] Schieber et al. (2017) Quantification of network structural dissimilarities. *Nat. Commun.* **8**, 13928.

[BB19] Bagrow & Boltt (2019) An information-theoretic, all-scales approach to comparing. . . *Appl. Netw. Sci.* **4**, 45.

model validation

(networks) national wars, Bitcoin transactions & Internet map
(models) war pact \gg small-world, scale-free & random graphs



(measure) portrait divergence \mathcal{P} [BB19]

[BB19] Bagrow & Boltt (2019) An information-theoretic, all-scales approach to comparing. . . *Appl. Netw. Sci.* 4, 45.

model structure

(size) model reproduces nodes n & edges m by design

(connectivity) model well reproduces giant component LCC

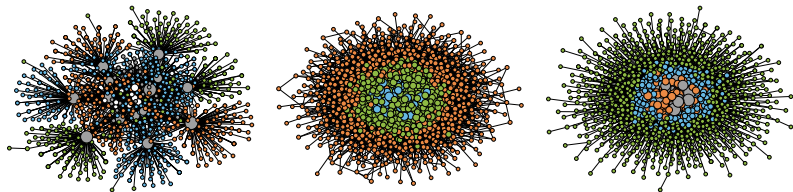
(distance) model well reproduces distance $\langle d \rangle$ & diameter d_{max}

	n	m	$\langle k \rangle$	LCC	$\langle C \rangle$	$\langle d \rangle$	d_{max}
Correlates of war	41	54	2.63	87.8%	0.28	2.58	8
	41	54	2.63	90.2%	0.06	2.64	7
International trade	130	3 730	57.38	100.0%	0.50	2.24	5
	130	3 730	57.38	100.0%	0.53	2.17	5
Bitcoin transactions	1 288	6 236	9.68	98.8%	0.33	2.83	9
	1 288	6 236	9.68	98.0%	0.13	3.08	7
Autonomous systems	3 213	11 248	7.00	100.0%	0.18	3.77	9
	3 213	11 248	7.00	98.3%	0.03	3.62	9

(clustering) model often underestimates node clustering $\langle C \rangle$

model **conclusions**

(**novel**) **simple model** of **networks that shrink**
(**others**) **in contrast** to classic **static** & **growing models**
(**networks**) model **well reproduces structure** except clustering



(**question**) **growing** or **shrinking models** more "**reasonable**"?
(**future**) **combined model**, other networks & analytical results

thank you!

arXiv:1909.00745v2

Naglić & Šubelj (2019) War pact model of shrinking networks. *PLoS ONE* 14(10), e0223480.

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