Structured-world conjecture: On modules and communities in real-world networks

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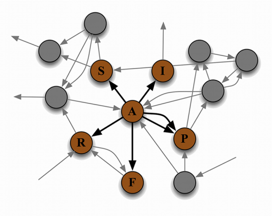
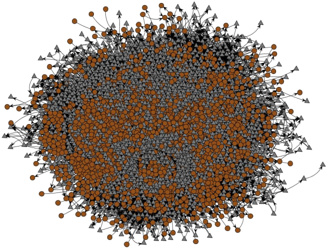
Outline

Motivation

Are there modules that could explain the structure of software networks?

```
class A extends S implements I {

  F field;

  public A (P parameter) {
    ...
  }

  public R function(P parameter) {
    ...
    return R;
  }

}
```





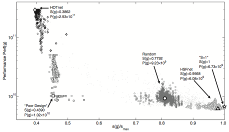
Outline

Degree mixing

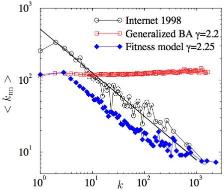
- Degree mixing coefficient $r \in [-1, 1]$. (Newman [30])

$$r = \frac{1}{2m\sigma_k} \sum_{ij} (k_i - k)(k_j - k),$$

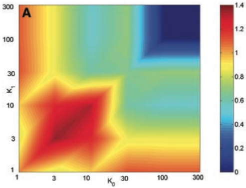
  where σ_k is the standard deviation and k_i degree of node i.
- Assortative mixing refers to $r > 0$, and disassortative to $r < 0$.
- r is simply a Pearson correlation coefficient of k_i at links' ends.



1) s-metric [23]



2) Γ connectivity [38]



3) Correlation profiles [27]

Degree mixing (II)

- Social networks are assortative, while most other are disassortative!

Type	Network	n	m	k	C	D	r
Collaboration	*netsci* [33]	1589	2742	3.5	0.638	0.690	0.462
	condmat [29]	27519	116181	8.4	0.655	0.722	0.166
	comsci [3]	239	568	4.8	0.479	0.561	−0.044
Online social	*pgp* [5]	10680	24316	4.6	0.266	0.317	0.238
Social	*football* [11]	115	613	10.7	0.403	0.419	0.162
	jazz [12]	198	2742	27.7	0.617	0.703	0.020
	dolphins [25]	62	159	5.1	0.259	0.319	−0.044
	karate [58]	34	78	4.6	0.571	0.666	−0.476
Communication	*emails* [14]	1133	5451	9.6	0.220	0.253	0.078
	enron [20]	36692	183831	10.0	0.497	0.530	−0.111
Road network	*euro* [50]	1039	1305	2.5	0.019	0.025	0.090
Power grid	*power* [56]	4941	6594	2.7	0.080	0.100	0.003
Citation	*hepart* [1]	27770	352285	25.4	0.312	0.353	−0.030
Documentation	*javadoc* [49]	2089	7934	7.6	0.373	0.433	−0.070
Protein	*yeast1* [37]	2445	6265	5.1	0.215	0.250	−0.101
	yeast2 [15]	2114	2203	2.1	0.059	0.072	−0.162
Software	*javax* [53]	1595	5287	6.6	0.381	0.440	−0.120
	jung [53]	317	719	4.5	0.366	0.423	−0.190
	guava [54]	174	355	4.1	0.320	0.375	−0.218
	java [53]	1516	10049	13.3	0.685	0.731	−0.283
Web graph	*blogs* [2]	1490	16715	22.4	0.263	0.293	−0.221
Metabolic	*elegans* [16]	453	2025	8.9	0.646	0.710	−0.226
Internet	*oregon* [20]	767	1734	4.5	0.293	0.317	−0.299
Bipartite	*women* [8]	32	89	5.6	0.000	0.000	−0.337

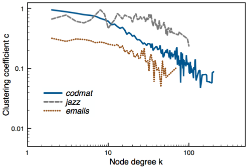
Network clustering

- Network clustering coefficient $C = \frac{1}{n}\sum_i c_i$. (Watts and Strogatz [56])

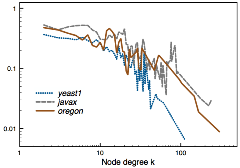
$$c_i = \frac{t_i}{\binom{k_i}{2}},$$

where t_i is number of links among Γ_i, $c_i \in [0,1]$.

- For many real-world networks $c_i \sim 1/k_i$. [41, 42, 48]



4) Degree assortative



5) Degree disassortative

- High degree nodes never have high c_i!

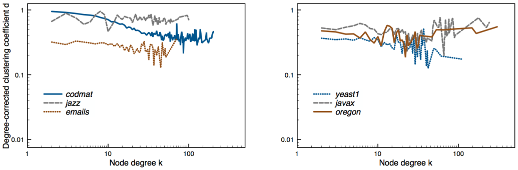
Degree-corrected clustering

- Network degree-corrected clustering co. $D = \frac{1}{n}\sum_i d_i$. (Soffer and Vázquez [46])

$$d_i = \frac{t_i}{\omega_i},$$

where ω_i is the max. number of links with respect to $\{k_i\}$, $d_i \in [0,1]$.

- Since $\omega_i \leq \binom{k_i}{2}$, $d_i \geq c_i$ and $D \geq C$ by definition.

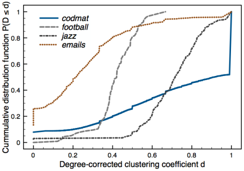


6) Degree assortative 7) Degree disassortative

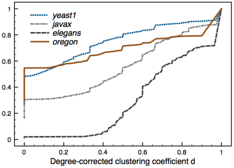
- For pseudo-fractal model $c_i \sim 1/k_i$ implies $c_i \sim 1/\log k_i$. [46]

Degree-corrected clustering (II)

- Most nodes in assortative networks share similar $d_i \gg 0$, whereas 30-55% of nodes in disassortative networks have $d_i \approx 0$!

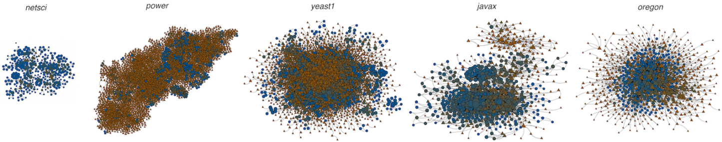


8) Degree assortative



9) Degree disassortative

- d_i appear to capture certain characteristics of the underlying domain.



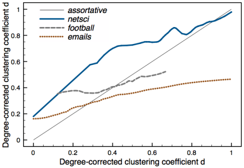
Clustering mixing

- Define clustering mixing coefficients $r_c, r_d \in [-1, 1]$. (Šubelj and Bajec [54])

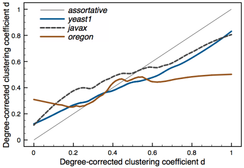
$$r_d = \frac{1}{2m\sigma_d} \sum_{ij} (d_i - D)(d_j - D),$$

where σ_d is the standard deviation. (Similarly for r_c.)

- Contrary to r_c, $r_d \gg 0$ in real-world networks!



10) Degree assortative



11) Degree disassortative

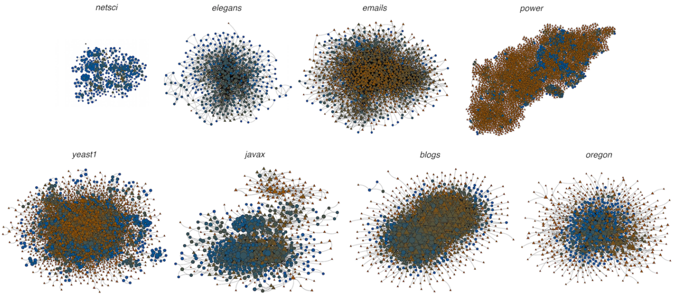
Clustering mixing (II)

Type	Network	n	m	k	C	D	r	r_c	r_d	$d_i < p_r$	$d_i < p_c$
Collaboration	*netsci* [33]	1589	2742	3.5	0.638	0.690	0.462	0.442	0.679	1%	1%
	condmat [29]	27519	116181	8.4	0.655	0.722	0.166	0.116	0.291	1%	1%
	comsci [3]	239	568	4.8	0.479	0.561	−0.044	0.123	0.355	6%	6%
Online social	*pgp* [5]	10680	24316	4.6	0.266	0.317	0.238	0.497	0.632	27%	27%
Social	*football* [11]	115	613	10.7	0.403	0.419	0.162	0.369	0.385	0%	0%
	jazz [12]	198	2742	27.7	0.617	0.703	0.020	0.008	0.198	1%	1%
	dolphins [25]	62	159	5.1	0.259	0.319	−0.044	0.192	0.234	15%	15%
	karate [58]	34	78	4.6	0.571	0.666	−0.476	−0.229	0.277	3%	6%
Communication	*emails* [14]	1133	5451	9.6	0.220	0.253	0.078	0.214	0.317	14%	15%
	enron [20]	36692	183831	10.0	0.497	0.530	−0.111	0.185	0.379	4%	4%
Road network	*euro* [50]	1039	1305	2.5	0.019	0.025	0.090	0.395	0.499	91%	91%
Power grid	*power* [56]	4941	6594	2.7	0.080	0.100	0.003	0.469	0.653	74%	74%
Citation	*hepart* [1]	27770	352285	25.4	0.312	0.353	−0.030	0.132	0.370	6%	6%
Documentation	*javadoc* [49]	2089	7934	7.6	0.373	0.433	−0.070	0.090	0.440	9%	9%
Protein	*yeast1* [37]	2445	6265	5.1	0.215	0.250	−0.101	0.372	0.534	29%	29%
	yeast2 [15]	2114	2203	2.1	0.059	0.072	−0.162	0.576	0.675	68%	68%
Software	*javax* [53]	1595	5287	6.6	0.381	0.440	−0.120	−0.041	0.545	17%	17%
	jung [53]	317	719	4.5	0.366	0.423	−0.190	0.092	0.443	21%	21%
	guava [54]	174	355	4.1	0.320	0.375	−0.218	0.075	0.734	34%	34%
	java [53]	1516	10049	13.3	0.685	0.731	−0.283	−0.574	0.536	1%	100%
Web graph	*blogs* [2]	1490	16715	22.4	0.263	0.293	−0.221	−0.057	0.308	8%	13%
Metabolic	*elegans* [16]	453	2025	8.9	0.646	0.710	−0.226	−0.240	0.183	1%	3%
Internet	*oregon* [20]	767	1734	4.5	0.293	0.317	−0.299	−0.231	0.262	35%	70%
Bipartite	*women* [8]	32	89	5.6	0.000	0.000	−0.337			100%	100%

$p_r = \frac{k}{n-1}$ and $p_c \leq \frac{(\sum_i k_i^2 - nk)^2}{n^3 k^3}$, while percentages ignore nodes with $k_i \leq 1$.

Clustering assortativity

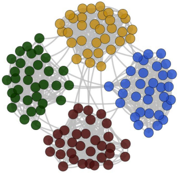
- $r_d \gg 0$ in real-world networks! ($r_c < 0$ in disassortative networks.)
- $d_i \approx 0$ and $r_d \gg 0$ imply connected regions with no clustering.



- r_d captures how well separated are different network structures.
- $r_d \nrightarrow 0$ when $n \to \infty$ in a random graph, however, $D \approx 0$.

Network structures

- Let community be a densely linked group of nodes that are sparsely linked with the rest of the network.
  - Consequence of homophily [28, 34] or triadic closure [13] in social networks.
  - Result in degree assortativity, when their sizes differ. (Newman and Park [36])
- Recently, communities are a consequence of clustering. (Foster et al. [10])
- There is substantial evidence that communities appear concurrently with high clustering and assortative mixing by degree. [31, 21, 57]



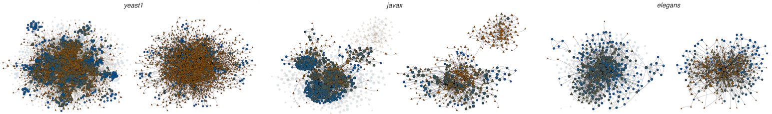
- Non-social real-world networks greatly deviate from this picture!

Network structures (II)

- Most real-world networks still contain at least some communities.
- Community extraction: (Zhao et al. [59])
  1. generate a pool of candidate communities,
  2. extract community S with the highest value of W,

$$W = s(n-s)\left(\frac{\sum_{i \in S} k_i^S}{s^2} - \frac{\sum_{i \in S} k_i - k_i^S}{s(n-s)}\right),$$

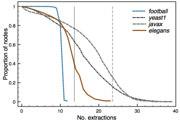
     where k_i^S and $k_i - k_i^S$ are internal and external degree of node i.
  3. repeat step 1. until W drops below the value expected at random.
- Extract only the links within S, but not those between S and S^C!



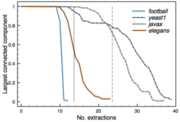
Communities overlaid over original networks and networks after extraction, respectively.

Network structures (III)

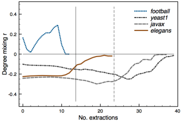
- After extraction of communities ≈ 80% nodes remain!
- Network structure beyond communities is characterized by:
  - disassortative mixing by degree,
  - lower (degree-corrected) clustering,
  - short distances between the nodes.



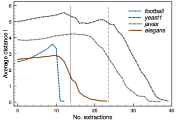
12) # nodes n



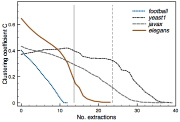
13) LCC



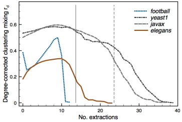
14) Mixing r



15) Distances l



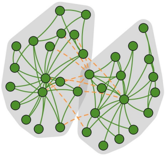
16) Clustering C



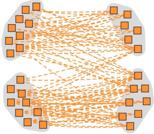
17) Mixing r_d

Network structures (IV)

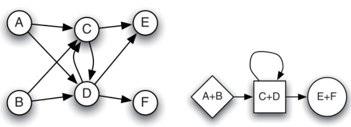
- Are there mesoscopic structures that could explain these properties?
- Let a module be a group of nodes with common neighbors.



18) Communities



19) Modules

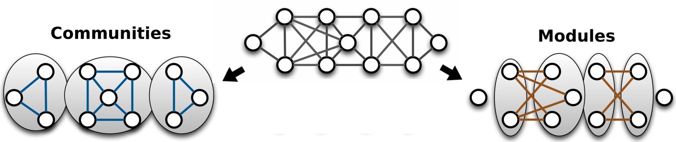


20) Role models [43]

- Modules coincide with groups of regularly equivalent nodes.
- Such modules should result in:
  - disassortative mixing by degree, as long as their sizes differ,
  - lower (degree-corrected) clustering (absence of triangles),
  - short distances between the nodes (efficient global navigation).

Structured-world conjecture

- Structured-world conjecture:
  Real-world networks are composed of modules characterizing different functions (roles) within the system and overlaid by communities based on some assortative tendency of the nodes, and noise.



- Modules explain degree disassortativity and efficient long-range navigation, whereas communities increase overall clustering and degree assortativity, and explain efficient short-range navigation.
- Structured-world networks must necessarily be heterogeneous!

Note that degree disassortativity and low clustering are already expected properties of scale-free networks.

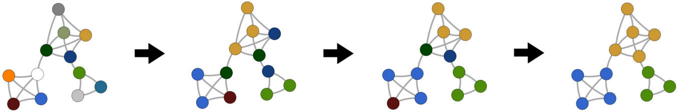
Outline

Label propagation

- Let g_i be unknown node (module) labels.
- Label propagation algorithm (LPA): (Raghavan et al. [40])
  1. initialize nodes with unique labels, $g_i = i$,
  2. node i adopts the label shared by most in Γ_i,

$$g_i = \operatorname*{argmax}_g \sum_{j \in \Gamma_i} \delta(g_j, g),$$

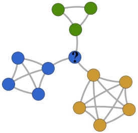
  3. repeat step 2. until convergence.



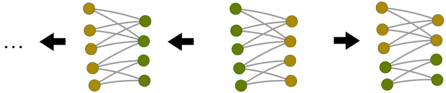
- Algorithm has near linear time complexity $\mathcal{O}(m^{1.2})$. (Šubelj and Bajec [51])

Label propagation (II)

- Convergence issues for, e.g., overlapping communities.
  $\hookrightarrow$ g_i is retained, when among most frequent in Γ_i.



- Oscillation of labels in, e.g., bipartite networks.
  $\hookrightarrow$ g_i are updated in a random order (sequentially).



- Results can be improved by applying node preferences f_i. (Leung et al. [22])

$$g_i = \operatorname*{argmax}_g \sum_{j \in \Gamma_i} f_j \cdot \delta(g_j, g)$$

Balanced propagation

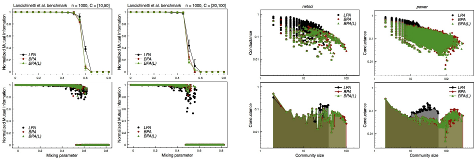
- Balanced propagation algorithm (BPA): (Šubelj and Bajec [50])

$$g_i = \operatorname*{argmax}_g \sum_{j \in \Gamma_i} b_j \cdot \delta(g_j, g),$$

where $b_i = \frac{1}{1+e^{-\eta(i_i-\lambda)}}$ (or $b_i = i_i$) and i_i is index of i, $i_i \in (0,1]$.

- Algorithm retains scalability, and improves stability and performance.

Algorithm	# distinct in 1000 partitions					
	karate	*dolphins*	*books*	*football*	*jazz*	*elegans*
LPA	184	525	269	414	63	707
BPA	19	36	29	154	20	75

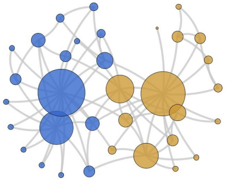


Defensive propagation

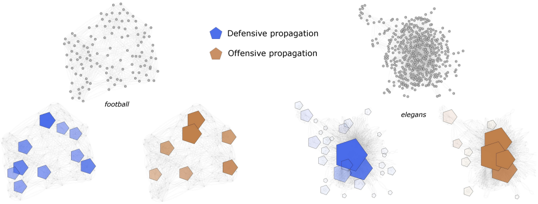
- Defensive propagation algorithm (DPA): (Šubelj and Bajec [51])

$$g_i = \underset{g}{\operatorname{argmax}} \sum_{j \in \Gamma_i} p_j \cdot \delta(g_j, g),$$

where p_i is the probability of a random walker utilized on g_i.



23) Community cores



24) Defensive and offensive propagation

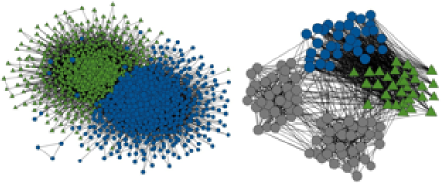
- Defensive and offensive prop. obtain high "recall" and "precision".

General propagation

- Label propagation can detect only connected (cohesive) structures.
- For modules, labels can be propagated through common neighbors!
- General propagation algorithm (GPA): (Šubelj and Bajec [55])

$$g_i = \underset{g}{\operatorname{argmax}} \left(\nu_g \sum_{j \in \Gamma_i} f_j \cdot \delta(g_j, g) + (1 - \nu_g) \sum_{j \in \Gamma_i} \sum_{l \in \Gamma_j \setminus \Gamma_i} \tilde{f}_l / k_j \cdot \delta(g_l, g) \right),$$

where $\nu_g \in [0, 1]$ are parameters and $f_i = b_i p_i$ (similarly for $\tilde{f}_i$).

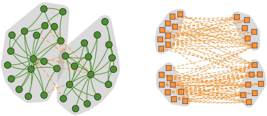


- ν_g are ≈ 1 and ≈ 0 for communities and modules, respectively.

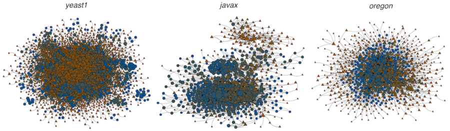
General propagation (II)

- Modeling of ν_g is of vital importance (guides the algorithm).
  - Dynamic based on conductance Φ. (Šubelj and Bajec [55])
  - Dynamic based on clustering C. (Šubelj and Bajec [52])
- Simple model based on clustering D (and mixing r_d): (Šubelj and Bajec [54])

$$\nu_{g_i} = \begin{cases} 1 & \text{for } d_i \geq p_c \ (D \geq p_c), \\ 0 & \text{for } d_i < p_c \ (D < p_c), \\ 0.5 & \text{otherwise.} \end{cases}$$



25) $d_i \geq p_c$ or $d_i < p_c$.

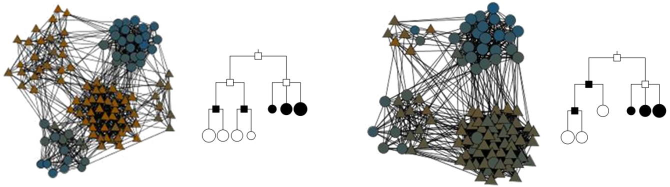


26) $d_i \geq p_c$ and $d_i < p_c$!

- Model seems to ignore most modules (structured-world conjecture)!

Hierarchical propagation

- k-partite network on n nodes becomes a clique when $k \rightarrow n$ or $n \rightarrow k$.
- Modules can become obscure in the presence of communities!
- How community detection algorithms identify network modules?

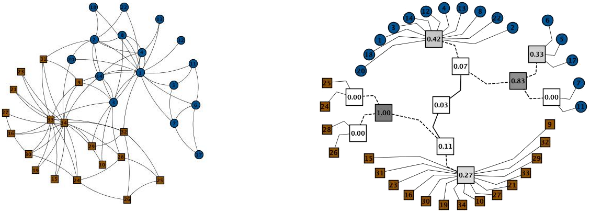


$\hookrightarrow$ Dependent modules can be identified as a community, and refined.

- Note that modules must be detected "twice"!

Hierarchical propagation (II)

- Hierarchical propagation algorithm (HPA): (Šubelj and Bajec [54])
  1. partition the network into communities and modules using GPA,
  2. refine each module (step 1.) and accept refinements that increase $\mathcal{L}$,
  3. repeat step 1. on a super-network induced by initial structures.
- Algorithm reveals entire hierarchy $\mathcal{H}$, where $\mathcal{L}$ is the likelihood of $\mathcal{H}$.



Bottom-most level of $\mathcal{H}$ is reported for structure detection.

- Time complexity for each level of $\mathcal{H}$ can be estimated to $\mathcal{O}((km)^{1.2})$.

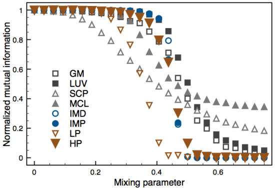
Hierarchical propagation (III)

- Single algorithm for communities and modules.
- No prior knowledge is required (e.g., number of structures)!
- Algorithm uses only local information (parallelization).
- Relatively simple to extend (e.g., prior knowledge).
- Time complexity is near ideal $\mathcal{O}(km)$!
- Relatively simple to implement.

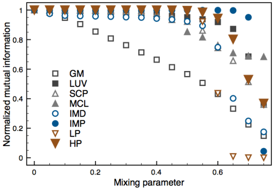
Outline

Community detection

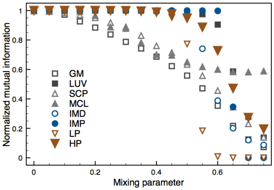
Community detection algorithms: greedy modularity [32, 6] (GM), multi-stage modularity [4] (LUV), sequential clique percolation [18] (SCP), Markov clustering [47] (MCL), Infomod [45] (IMD), Infomap [44] (IMP), label propagation [40] (LP) and hierarchical propagation [54] (HP).



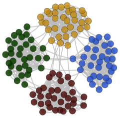
27) (Girvan and Newman [11])



28) (Lancichinetti et al. [19]) (small)

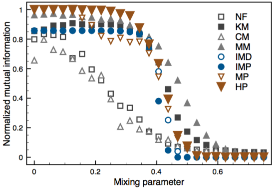


29) (Lancichinetti et al. [19]) (big)

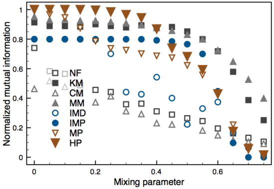


Module detection

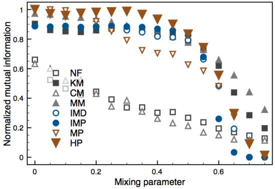
Module detection algorithms: matrix factorization [9] (NF), *k*-means [26] based on [24] (KM), mixture model [35] (MM), degree-corrected mixture model [17] (CM), Infomod [45] (IMD), Infomap [44] (IMP), model propagation [52] (MP) and hierarchical propagation [54] (HP).



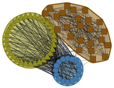
30) (Pinkert et al. [39])



31) (Šubelj and Bajec [54]) (HN6)



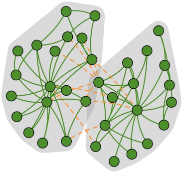
32) (Šubelj and Bajec [54]) (HN7)

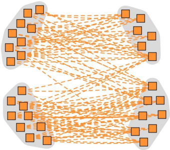


Real-world networks

Structure detection algorithms: multi-stage modularity [4] (LUV), mixture model [35] (MM), classical propagation [54] (CP) and hierarchical propagation [54] (HP).

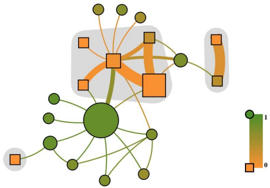
Network	NMI				ARI			
	LUV	MM	CP	HP	LUV	MM	CP	HP
football	0.876	0.823	0.905	**0.909**	0.771	0.683	0.841	**0.850**
karate	0.629	**0.912**	0.834	0.866	0.510	**0.912**	0.823	0.861
jung	0.605	0.662	0.650	**0.684**	0.269	0.276	0.218	**0.280**
women	0.309	0.825	0.217	**0.932**	0.174	0.716	0.119	**0.936**



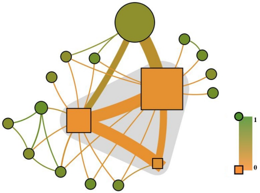


33) Zachary karate net. 34) Davis women net.

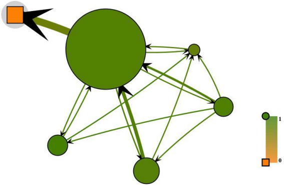
Real-world networks (II)



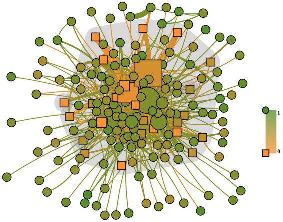
35) jung software network



36) javax software network



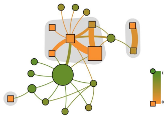
37) Amazon web graph



38) Protein interactions

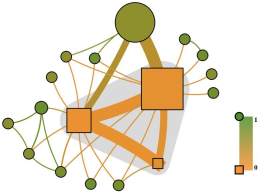
Real-world networks (III)

Network	Module	n	$1-\Phi$	Description
	Core community	65	0.86	[jung.visualization.] *(Server|Viewer|Pane|Model|Context) (9); control.* (4) control.*Control (5); layout.* (7); picking.*State (3); picking.*Support (6); renderers.*Renderer (13); renderers.*Support (3); etc.
	5-conf. (upper left)	3	0.00	[jung.algorithms.filters.] *Filter (3).
	5-conf. (upper right)	21	0.33	[jung.graph.] *(Graph|Multigraph|Tree) (18); etc.
jung	5-conf. (central)	28	0.07	[jung.] algorithms.generators.*Generator (2); algorithms.importance.* (4) algorithms.layout.*Layout* (3); algorithms.scoring.*Scorer (2); algorithms.shortestpath.* (2); graph.*(Graph|Tree|Forest) (4); etc. (interfaces)
	5-conf. (lower left)	13	0.00	[jung.algorithms.] layout.*Layout* (7); layout3d.*Layout (3); etc.
	5-conf. (lower right)	44	0.03	[jung.] algorithms.cluster.*Clusterer* (4); algorithms.generators.random.*Generator (5); algorithms.importance.*Betweenness* (3); algorithms.metrics.* (3); algorithms.scoring.** (5); algorithms.shortestpath.* (5); graph.util.* (7); etc. (implementations)
	2-conf. (upper)	13	0.03	[jung.io.graphml.] parser.*Parser (10); etc.
	2-conf. (lower)	13	0.38	[jung.io.graphml.] *Metadata (8); etc.
	1-conf. (central)	2	0.00	[jung.visualization.control.] *Plugin (2).



Real-world networks (IV)

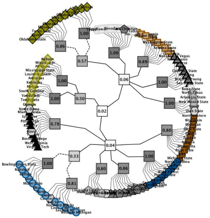
Network	Module	n	$1-\Phi$	Description
javax	Core community	179	0.64	[javax.swing.] plaf.*UI (24); plaf.basic.Basic*UI (42); plaf.metal.Metal*UI (22); plaf.multi.Multi*UI (30); plaf.synth.Synth*UI (40); etc.
	3-conf. (upper)	193	0.15	[javax.] accessibility.Accessible* (10); swing.J* (41); swing.**(Border|Borders|Box|Button|Dialog|Divider|Editor|Factory|Filter|Icon|Kit|LookAndFeel|Listener|Model|Pane|Panel|Popup|Renderer|UIResource|View) (92); etc.
	3-conf. (left)	113	0.11	[javax.] accessibility.Accessible* (6); swing.* (34); swing.event.*Event (8); swing.event.*Listener (13); swing.plaf.*UI (6); etc.
	3-conf. (lower)	44	0.19	[javax.swing.] text.*View (15); text.html.*View (16); etc.

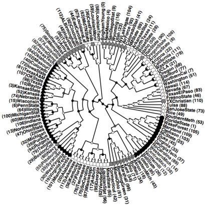


Structure prediction

- How well the model fits the network observed? Not link prediction!

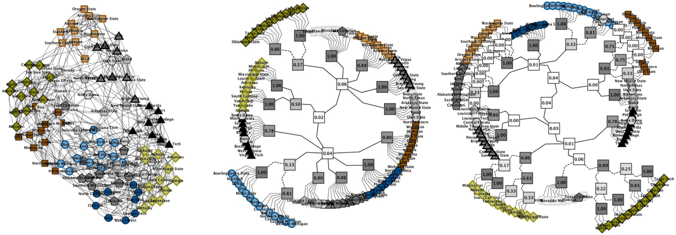
Network	Runs	$-\log \mathcal{L}$ and # levels CP		HP—p_r		and p_c		(Clauset et al. [7])	
football	10^4	1010.9	3	**954.8**	**5**	1004.1	3	*884.2*	*11*
karate	10^5	174.1	3	**172.3**	**3**	173.9	2	*73.3*	*10*
euro	10^3	4108.9	6	**3883.2**	**8**	3924.4	5		
yeast2	10^2	12495.0	6	11611.2	7	**11596.4**	**4**		
javax	10^2	13020.7	4	12894.1	4	**11512.2**	**3**		
jung	10^3	2354.5	5	2312.5	4	**2272.9**	**4**		
elegans	10^2	8734.1	5	8640.9	6	**8243.3**	**5**		
women	10^4	193.9	2	**163.6**	**1**	**163.6**	**1**		



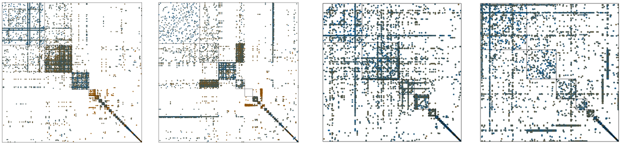


39) Module hierarchy 40) Binary hierarchy

Structure prediction (II)



Hierarchies revealed with CP and HP algorithms, respectively.

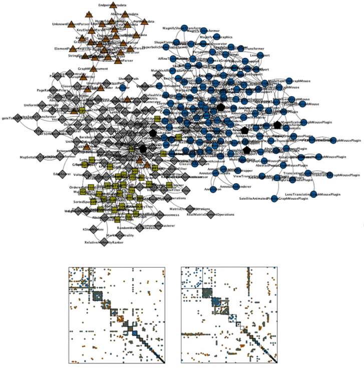
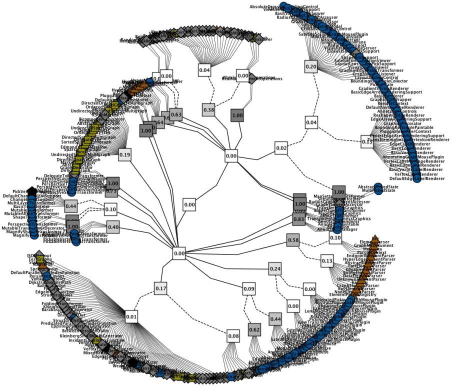


41) javax software network   42) elegans metabolic network

Hierarchies and blockmodels revealed with CP and HP algorithms, respectively.

Software networks

- Software network structures coincide with software packages.
- Communities and modules more accurately predict packages than communities alone!

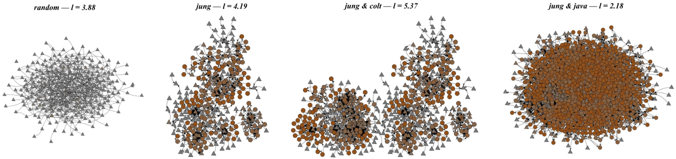
Blockmodels revealed with CP and HP algorithms, respectively.

Software networks (II)

- Software packages can be predicted with $\approx 80\%$ accuracy, whereas complete hierarchy can be precisely identified for over 60% of classes!

			CA				
Network	l	l_∞	P	P_4	P_3	P_2	P_1
flamingo	2.65	4	**0.566**	$\leftarrow$	0.572	*0.793*	1.000
colt	3.35	4	**0.654**	$\leftarrow$	*0.756*	0.942	1.000
jung	2.97	4	**0.617**	$\leftarrow$	0.663	*0.857*	1.000
org	3.50	7	**0.616**	0.616	*0.714*	0.989	1.000
weka	3.02	6	**0.684**	0.692	*0.736*	0.871	1.000
javax	3.11	5	**0.626**	0.631	*0.816*	0.982	1.000

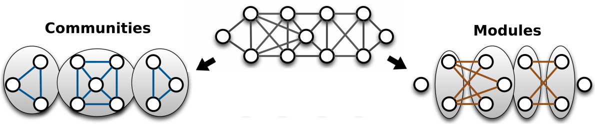
- Networks should not be combined with the core of the language.



Outline

Conclusions

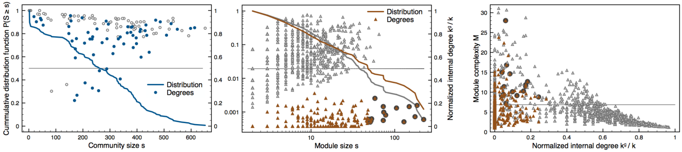
- Structured-world conjecture provides a mesoscopic view on the structure of real-world networks!
  ↪ Different structures imply different macroscopic network properties.
  ↪ Clustering assortativity captures how different modules are merged.
  ↪ Conjecture combines scale-free and small-world phenomena.



- Parameter-free algorithm for detection of communities and modules.
  ↪ Algorithm is (at least) comparable to current state-of-the-art.
  ↪ Network properties could be further utilized within the algorithm!

Future work

- *How do dependent modules link between each other?*
  ↪ Necessary to develop a measure of module quality.
- Results suggest that module complexity is much larger than expected!



- *How to utilize degree mixing within the algorithm?*
  ↪ Necessary to analyze networks with millions (billions) of nodes.

Thank you.

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www http://lovro.lpt.fri.uni-lj.si/

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